AMMRC TR 78-44

AD

DETERMINATION OF FRACTURE STRESS AND EFFECTIVE CRACK TIP RADIUS FROM TOUGHNESS (K_{Ic}) AND YIELD STRENGTH (Y)

October 1978

ARMY MATERIALS AND MECHANICS RESEARCH CENTER
Watertown, Massachusetts 02172

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
AMMRC TR 78-44		

4. TITLE (and Subtitle)

DETERMINATION OF FRACTURE STRESS AND EFFECTIVE CRACK TIP RADIUS FROM TOUGHNESS (K_{Ic}) AND YIELD STRENGTH (Y)

5. TYPE OF REPORT & PERIOD COVERED

Final Report

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)

Reinier Beeuwkes, Jr., Anna M. Hansen, and Robert T. Tremblay*

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS

Army Materials and Mechanics Research Center
Watertown, Massachusetts 02172
DRXMR-TM

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

D/A Project: 1L161102AH42
AMCMS Code: 611102.H420011

11. CONTROLLING OFFICE NAME AND ADDRESS

U. S. Army Materiel Development and Readiness Command, Alexandria, Virginia 22333

12. REPORT DATE

October 1978

13. NUMBER OF PAGES

50

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)

Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

*Engineering Student Trainee

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fracture mechanics
Strength
Mechanical properties
Fracture toughness

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

(SEE REVERSE SIDE)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Block No. 20

ABSTRACT

Extensive analysis was completed of available fracture mechanics and strength data and graphical procedures developed to relate effective crack tip radius to inherent materials properties. Fracture stress and effective crack tip radius are estimated from a log-log plot of K_{Ic} and corresponding yield strength using transparent overlays supplied with the paper. Examples are employed to illustrate the method. The influence of testing temperature, metallurgical structure, chemistry, and the existence of an effective crack tip radius are discussed. Associated tables are included relating two dimensionless parameters involving toughness, yield strength, crack tip radius, and peak stress at any elasticity containing a degree of yielding consistent with valid toughness data.

CONTENTS

INTRODUCTION

This paper* is primarily a "how to" paper and is particularly devoted to explaining (Examples 1 and 2) and illustrating a simple graphical method which the writers have devised and used at AMMRC to determine fracture stress and an effective crack tip radius associated with K_{Ic} crack toughness failure, to replace the rather laborious cut-and-try method used in "Characteristics of Crack Failure,"[1] and to add visual clarity into the matching process employed there in so doing. However, the results obtained in Example 3 imply metallurgical mechanisms for the fracture strength and radius. Also, Example 4 quantitatively indicates how chemistry may affect toughness and by the character of its effect may imply the existence of a parameter such as the effective crack tip radius constituent of toughness employed herein, once it is supposed that altering chemistry implies altering a mechanical mechanism.

Tables are also presented which expand those given in the referenced paper and from which graphical overlays may be constructed for particular cases as desired.

Background for the method is supplied in the next section, but need not be read to follow the cases illustrated in the section entitled "Examples."

BACKGROUND

1. Crack Stress Analysis

Beeuwkes proposed and illustrated (see Reference 1) methods of determining fracture stress and crack tip radius at failure from measurements of plane strain fracture toughness K_{Ic} and corresponding yield strength values. The methods were based on the presumption that K_{Ic} toughness failure occurred when the (essentially) nil ductility fracture stress of the material used was reached, provided that this fracture prolonged the crack. A fracture stress failure across the crack, on the other hand, would block lengthening of the crack. Both types of failure were accounted for, however, depending on the relative magnitudes of the transverse and longitudinal fracture stresses of the material being tested. We are here concerned with the K_{Ic} type. The fracture stresses involved are essentially the so-called "nil ductility" type, since they are expected to occur close to the elastic-plastic boundary of yielding, ahead of, and a bit to either side of an imagined prolongation of the crack. Their magnitude for fracture is reached by the addition of a hydrostatic tension to the yield strength of the material used, which hydrostatic component is equal to the yield strength multiplied by the change in angle $\nmid$ of a shear stress trajectory, somewhat as in plastic-rigid theory, since the shear stress trajectories giving rise to appreciable hydrostatic tensions traverse regions of nearly constant yield strength. The change in angle of those trajectories, which have the maximum angular change with given amounts of yielding, increases with the amount of yielding. Thus in this theory — in contrast to the widely accepted assumption that yielding relieves stress concentration — the stress leading to fracture increases with the amount of yielding. That is, the yielding increases until a trajectory traversing the yielding region occurs which corresponds to sufficient stress to

*Prepared as of June 1975. The continuing investigation of Crack Tip Radius and Shape of L versus $\nmid$ or F is not expected to modify the procedures and technical conclusions of this report.

1. BEEUWKES, R., Jr. *Characteristics of Crack Failure.* Surfaces and Interfaces, Syracuse University Press, v. II, 1968, p. 277-311.

cause fracture. Figure 1 of failure of a specimen of polycarbonate into which a notch of small radius had been machined supports this concept of failure.[2]

These trajectories are associated with a crack tip radius, or what is effectively a crack tip radius, existing at the moment of K_{Ic} fracture. Since the trajectory associated with failure is some distance from the tip, the exact shape of the tip is not essential to the shape of the trajectory even though, e.g., it be caused by irregularities in length of the crack associated with microstructural variations at the crack front, by grain size, by inclusions, by irregularity in yield zone shape, or by simple yielding, itself, associated with work-hardening. This list is not exhaustive, it is only suggestive of the variety of explanations that may be postulated. However, measurements of crack opening have been made for AMMRC which generally support[3] the existence of a radius.

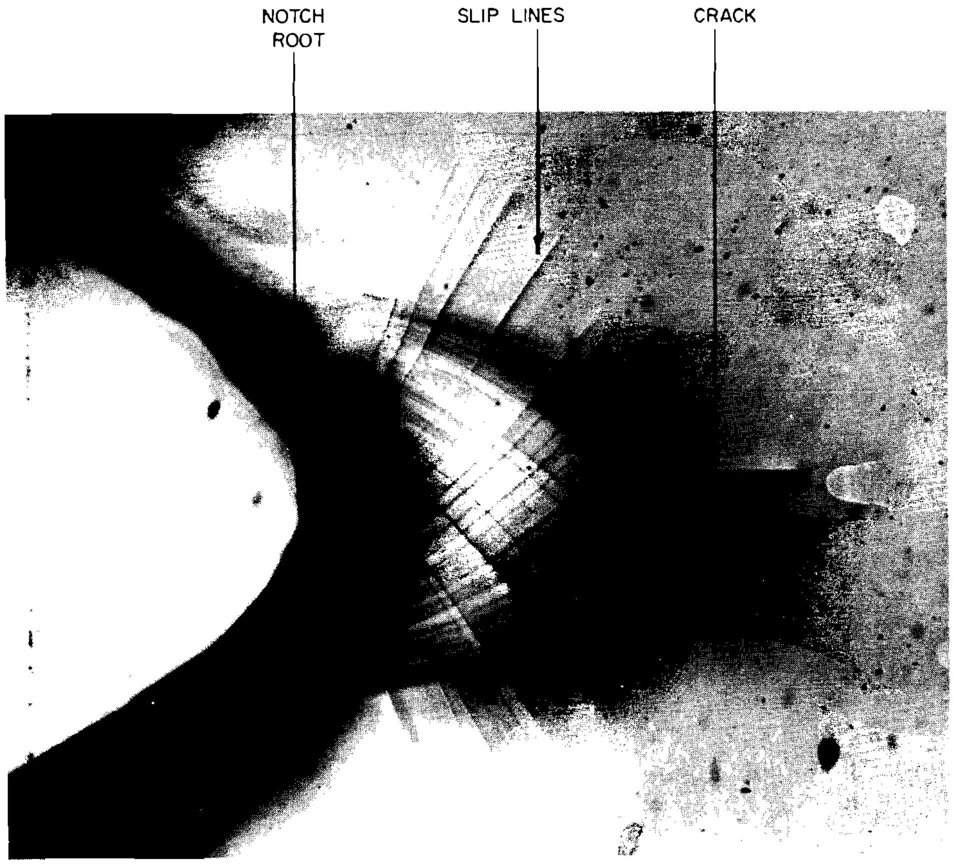


Figure 1. Transverse view of midthickness slice showing fracture origin at elastic-plastic boundary and slip lines. ρ = 0.005 in. Mag. 350X.

2. GARDE, A., and WEISS, V. *Brittle Fracture Initiation at the Elastic-Plastic Interface.* Syracuse University Research Institute, Contract DA-31-124-ARO(D)-112, Interim Technical Report, AMMRC CR 70-29, February 1970; also *Brittle Crack Initiation at the Elastic Plastic Interface,* Met. Trans. AIME, v. 3, November 1972, p. 2811.
3. TAGGART, R., and WAHI, K. K. *Crack Opening Displacement during the Tensile Loading of Ductile and Brittle Notched Plates.* University of Washington, Seattle, under subcontract to Syracuse University Prime Contract DA-31-124-ARO(D)-112, Final Report, AMMRC CTR 72-33, February 1972; also, abridged version entitled *The Relationship between the Fracture Toughness and the Crack Tip Radius* by R. Taggart, K. Wahi, and R. Beeuwkes, Jr., ASTM STP 605, 1976, p. 62.

The proposition that K_{Ic} depends on both fracture stress and radius leads to the presumption that both may possibly be individually alterable by metallurgical means and thus should be investigated.[4]

2. Material Fracture Behavior

The analysis of nominal stress S and trajectory angle change $\nmid$ of behavior described above led to a dimensionless relationship between $K = S\sqrt{\pi a}$, yield strength Y, crack tip radius at fracture ρ, Young's modulus of elasticity E, and Poisson's ratio μ, i.e.,

$$L = \frac{Y\sqrt{\pi\rho}}{K} + 2(1-\mu^2)\frac{Y}{E} = \text{function of } \nmid$$

where, if

F_1 = fracture stress parallel to S_{max}
F_2 = fracture stress parallel to S_{min}

then

$$F_1 = Y + Y\nmid$$
$$F_2 = Y\nmid$$

i.e.,

$$\nmid = (F_1 - Y)/Y \text{ or } \nmid = F_2/Y$$

if $F_2 < F_1 - Y$ and if F is parallel to S principal.

The modulus of elasticity term is often negligible. Also, as stated above, K_{Ic} failure corresponds to F_1.

This is for the center cracked plate under nominal tension stress S. A simple modification in terms of ordinary elastic stress concentration factor for notches of large depth to radius ratio a/ρ in other geometrical configurations is available in Reference 1.

Detailed tables of L versus $\nmid$ and $\nmid$ versus L are supplied in Appendix A.

There are two unknowns, F and ρ, in the above relationship for L, to be inferred from known values of K, Y, μ, and E or to be determined from separate considerations, especially directly from experiments.

The latter is sometimes possible, for example, nil ductility fracture strength for mild steel, especially for fracture across the transverse to rolling direction, where nil ductility fracture may be achieved by simply lowering the temperature. Crack tip radius may sometimes be inferred from sensitive measurements of crack opening displacement. Also, dynamic plate slap type experiments may be used for fracture stress if made with sufficiently low slap pressure and account is taken of reversal of stress. Such independent measurements are not only very important in themselves to elucidate or confirm their connection with toughness as in the

4. WEISS, V., and SANFORD, W. *Fracture Stress Determination Associated with Toughness and Brittle Fracture.* Syracuse University Contract DAAG46-73-C-0208, Final Report, AMMRC CTR 74-73, September 1974. (Work is attempt to check K_{Ic} inferred values of F by independent tension tests of fracture strength on the same material used by TRW in making K_{Ic} measurements reported in AMMRC CR 69-18, January 1971.)

theory above, but no measurements for such confirmation are known to exist. The measurements are especially important because, as discussed below, both fracture strength and crack tip radius at failure may possibly vary with yield strength in an unknown manner.

To infer fracture strength F and crack tip radius ρ from the L versus $\nless$ relationship and measurements of K and Y, two experimental points, e.g., K_1, Y_1, and K_2, Y_2, are all that are algebraically necessary, since there are but two unknowns, F and ρ, if it may be assumed that F and ρ do not vary with Y.

Expressed otherwise, these values of F and ρ are invalid in the region of K and Y unless use of them provides other analytically derived K and Y values which also lie on the experimental K versus Y curve, or unless choices of other pairs of experimental K,Y data for analysis of the region lead to the same values of F and ρ.

However, since the character of a curve at any point is governed by its radius of curvature, three data points in a region in which the curvature can be considered essentially constant also determine the character of the curve in the region and, if they are taken in pairs, lead to self-consistent F and ρ. If the three points all lie on the curve of the graphical methods below, then the fracture stress parameter and radius are known in that region, and the next adjacent region may be similarly examined to find applicable values. By continuation of this process curves of variation of the fracture stress parameters discussed below may be found, though these may not be constant over the whole region of interest. If no three point matches can be found, independent information must be resorted to.

a. Variable Test Temperature

Inference from general experience suggests that for steel, at least, the fracture stress of a given material may not vary significantly with test temperature, say from the temperature of liquid nitrogen to about four tenths of the absolute melting temperature, or with strain rate over a correspondingly large range. (Inferences for our purposes about effect of pulse duration in plate slap type experiments are complicated because of stress reversal and extraordinarily large pressures used and will not be discussed here.) Thus F is constant in this case and

$$F = Y + Y\nless = (1 + \nless)\ Y.$$

b. Variable Tempering Temperature

On the other hand, in the same test range as above, if the yield strength of a quenched material of given chemical composition is varied by changing the tempering temperature, the fracture stress is not constant but inference from general experience suggests that the fracture stress may be essentially a constant amount above the yield strengths, comparisons being made at any fixed temperature.

That is

$$F = Y + f = Y + Y\nless \text{ and}$$

$$f = Y\nless$$

where f is constant at any fixed testing temperature. Naturally, such a relationship cannot be expected to hold if there is retained austenite that varies in amount with tempering temperature.

EXAMPLES

The basis for very simple determination of fracture stress and crack tip radius at fracture by use of transparent overlays is given in Appendix B.

Such overlays are submitted with this report and their use is demonstrated for three cases:

1) F and ρ are not functions of Y.
2) $f = F - Y$ and ρ are not functions of Y.
3) Iterative correction for methods 1) and 2) possibly desirable if $(F - Y)/Y > 1.5$.

In addition, the use of the tables of L versus κ and κ versus L is illustrated in Example 4. Examples of the use of such tables to determine fracture strength and radius are given in Reference 1.

Example 1: F and ρ Are Not Functions of Y

Experimental yield strength and fracture toughness K_{Ic} data are shown in Figure 2a. It is plotted as K versus Y on single-cycle log-log paper as in Figure 2b. The "Constant F Overlay" is then positioned so that the curve best fits the data with the axes of the overlay parallel to the axes of the underlying graph paper as in Figure 3. The value of F is found where the abscissa = 1 line of the overlay crosses the Y axis. In this case, F = 476 ksi. The value of $F\sqrt{\pi\rho}$ is found where the ordinate = 1 line of the overlay crosses the K axis. In this case $F\sqrt{\pi\rho}$ = 24.5 ksi. Therefore, ρ = 0.00084 in. Points labeled "overlay points" in

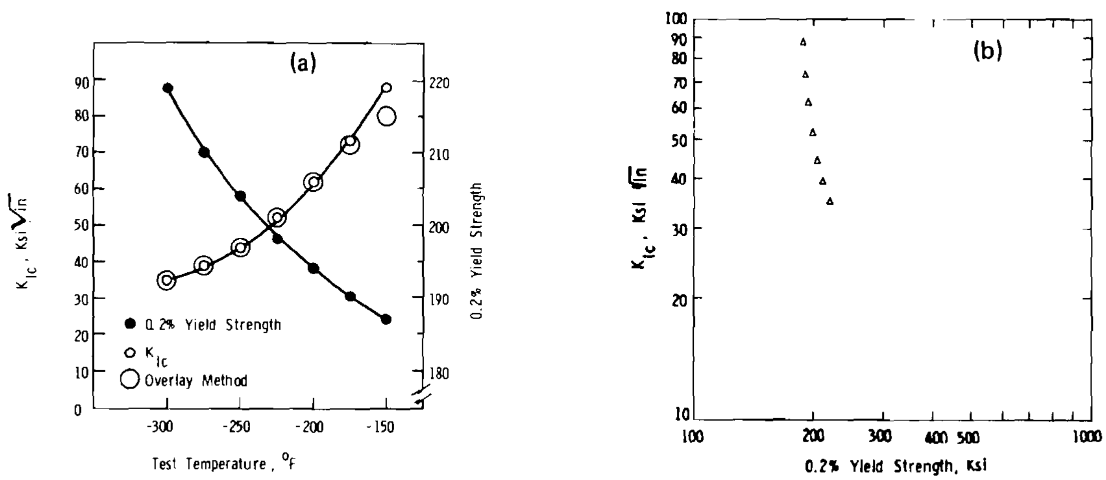


Figure 2. Experimental fracture toughness (K_{Ic}) and yield strength data is shown in (a). It is plotted as K versus Y on log-log paper as in (b) for use shown in Figure 3.

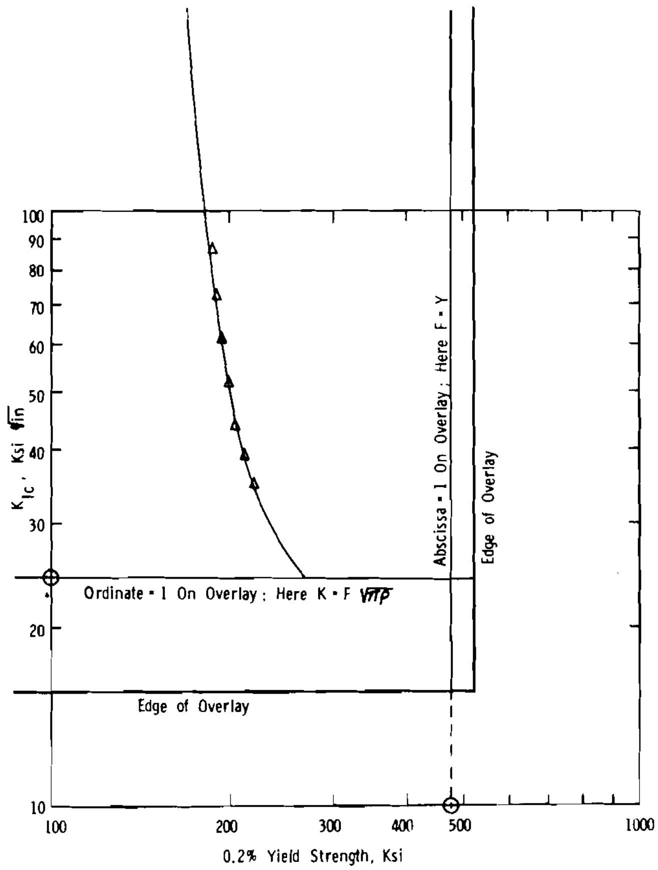


Figure 3. The Constant F Overlay is positioned so that the curve best fits the data of Figure 2b. $F\sqrt{\pi\rho}$ is the overlay unity ordinate intercept and F is the overlay unity abscissa intercept. Points under overlay curve are shown in Figure 2a.

Figure 2a were determined by reading the value of K_{Ic} from under the overlay curve in Figure 3 at the yield strengths of each of the original data points. Test data[5] are of 0.34% C, 0.71% Mn, 0.33% Si, 0.51% Cr, 3% Ni, 0.35% Mo, 0.12% V, 0.048% Al, 0.007% P, 0.009% S martensitic steel, heat 9.

Example 2: f = F - Y and ρ Are Not Functions of Y at Constant Test Temperature

Experimental yield strength and fracture toughness K_{Ic} data are shown in Figure 4a. It is plotted as K versus Y on single-cycle log-log paper, as in Figure 4b. The Constant f Overlay is then positioned so that the curve best fits the data with the axes of the overlay parallel to the axes of the underlying graph paper as in Figure 5. The value of f is found where the abscissa = 1 line of the overlay crosses the Y axis. In this case, f = 305 ksi. The value of 10 $f\sqrt{\pi\rho}$ is found where the ordinate = 10 line of the overlay crosses the K axis. In this case, 10 $f\sqrt{\pi\rho}$ = 146 ksi$\sqrt{\text{in.}}$ Therefore, ρ = 0.00073 in. "Overlay Method" points in Figure 4a were determined by reading the value of K_{Ic} from under the overlay curve in Figure 5 at the yield strengths of each of the original data points. Test data[6] are of 0.43% C, 0.64% Mn, 0.35% Si, 0.87% Cr, 3.25% Ni, 0.30% Mo, 0.10% V, 0.010% P, 0.004% S, 0.010% Al martensitic steel, heat 10, tested at -100 F.

5. VISHNEVSKY, C., and STEIGERWALD, E. A. *Influence of Alloying Elements on the Toughness of Low Alloy Martensitic High Strength Steels.* TRW Inc., Contract DAAG46-67-C-0171, AMMRC CR 68-09(F), November 1968.
6. VISHNEVSKY, C. *Effect of Alloying Elements on Tempered Martensite Embrittlement and Fracture Toughness of Low Alloy High Strength Steels.* TRW Inc., Contract DAAG46-69-C-0060, AMMRC CR 69-18(F), January 1971.

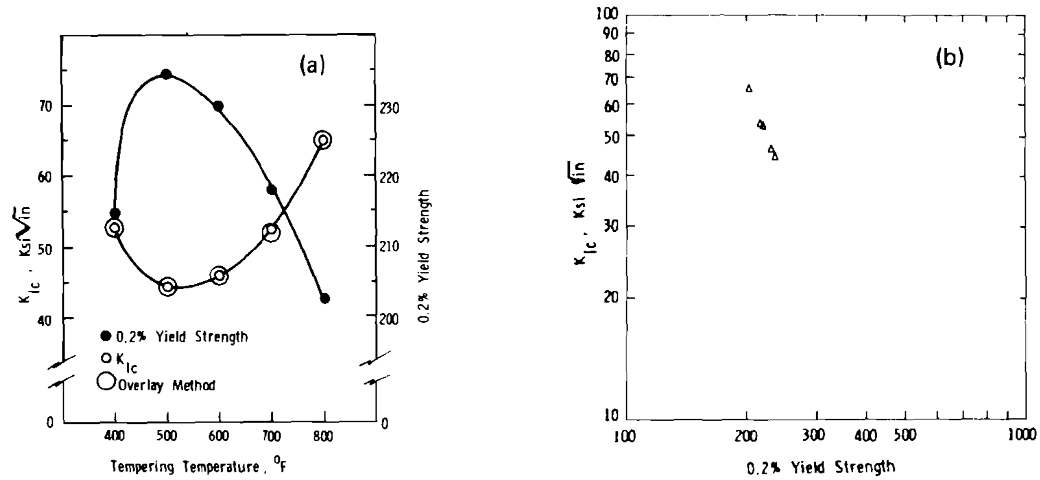


Figure 4. Experimental fracture toughness and yield strength data is shown in (a). It is plotted as K versus Y on log-log paper as in (b) for use in Figure 5.

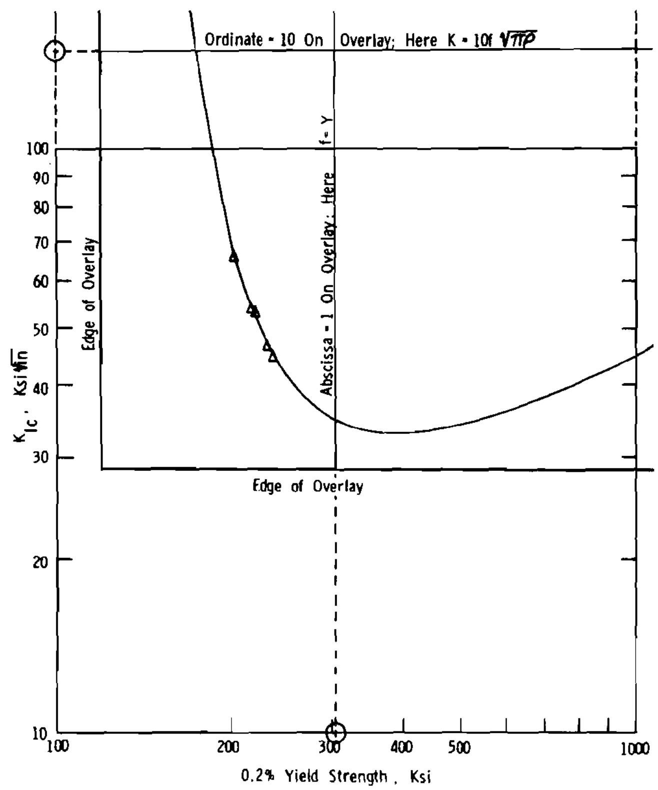


Figure 5. The Constant f Overlay is positioned so that the curve best fits the data of Figure 4b. $f\sqrt{\pi\rho}$ is the overlay unity ordinate intercept and f is the overlay unity abscissa intercept. Points under overlay curve are shown in Figure 4a.

Example 3: Iteration. Example: $f = F - Y$ and ρ Are Not Functions of Y at Constant Test Temperature

Here we show, by example, how to obtain more accurate values of f and ρ than can be obtained by the approximate method of Example 2. The use of this more accurate method, though simple, is only warranted if the experimental data is especially good and, more particularly, if for at least part of the data $(F-Y)/Y = f/Y > 1.5$. The corresponding method for F and ρ constant is very similar. The mathematical basis for the overlay method for constant f, F and other fracture criteria versus Y is given in Appendix B.

As shown in these appendixes we should plot

$$\log K' = \log \frac{K_{Ic}}{1 + 2(1-\mu^2)(K_{Ic}/E\sqrt{\pi\rho})} \text{ versus } \log Y$$

instead of experimental values of $\log K_{Ic}$ versus $\log Y$ as was done in the approximate method using the overlay. However, we do not know $\sqrt{\pi\rho}$ and therefore cannot exactly do this, but the term involving $\sqrt{\pi\rho}$ is small and therefore has a secondary effect on the plot. Thus we may assume $\sqrt{\pi\rho}$ has approximately the value obtained by the overlay method if we neglect the K_{Ic} term in the denominator of K' above, and use this $\sqrt{\pi\rho}$ value to compute K'. From the overlay and a plot of this log K' versus log Y we find a more exact value of $\sqrt{\pi\rho}$. We repeat the process as much as desired, always using the original data value of K_{Ic} and the latest value of $\sqrt{\pi\rho}$ in computing K'.

We will discuss matching and determination of f and ρ for all regions of Figure 6 which is the source of our experimental data,[7] and on which we have indicated our final results, but will reproduce our numerical work on iteration only for the three K_{Ic}, Y points corresponding to tempering temperatures of 600 F, 800 F, and 900 F.

First, neglecting the K term in the denominator of K', we proceed exactly as in Examples 1 and 2, fitting the f overlay through our three $\log K_{Ic}$ versus $\log Y$ data points, Figure 7, to determine f and ρ as follows:

Tempering Temperature deg F	Experimental Data		
	Y (ksi)	K_{Ic} (ksi$\sqrt{\text{in.}}$)	ɟ = f/Y
600	163.5	80.5	1.755
800	186.5	38	1.539
900	203.5	29	1.410

$f = 287$, $\quad f\sqrt{\pi\rho} = 7.8$, $\quad \sqrt{\pi\rho} = 0.0272$

Here the column ɟ = f/Y was added after determining f. We see from it that two values are greater than 1.5 and thus suppose that at least some iteration will be called for.

7. AULT, R. J., HOLTMAN, R. B., and MYERS, J. R. *Heat Treatment of a Cr-Mo-Co Martensitic Stainless Steel for Optimum Combinations of Strength, Toughness and Stress Corrosion Resistance.* ASM Trans. Quart. 61, no. 1, March 1968.

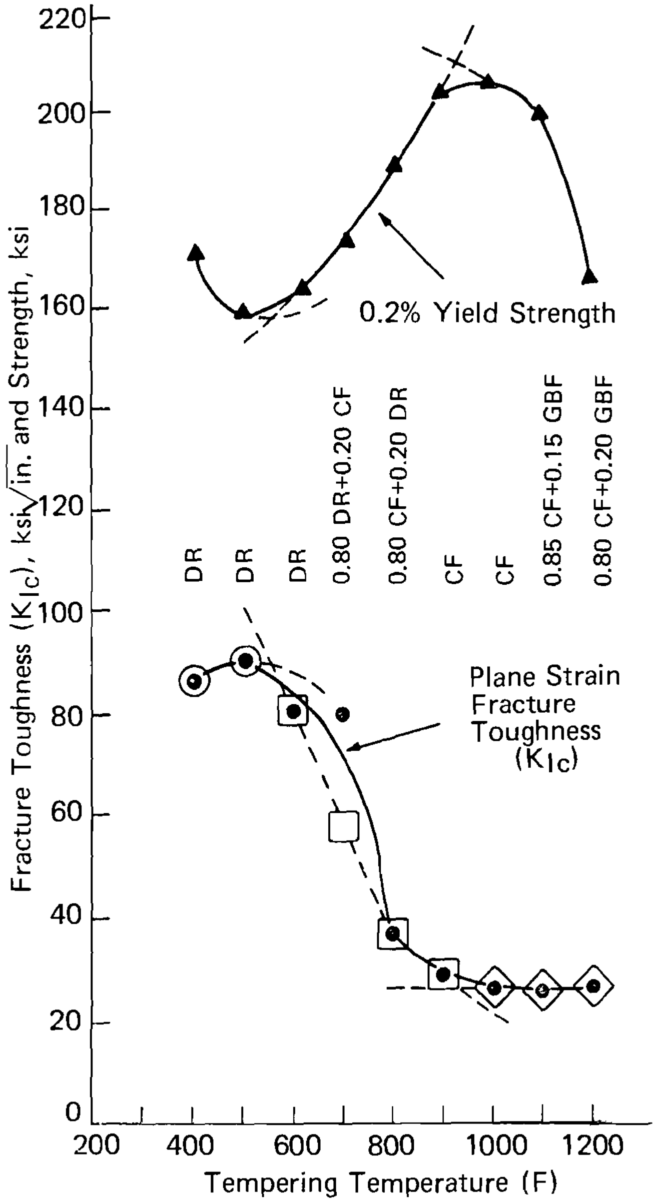


Figure 6. Effect of tempering temperature on the plane strain fracture toughness and yield strength of AFC-77 at room temperature (from Reference 7). K_{Ic} (circle, square and diamond) from yield strength Y and theory. For circle, radius = 1.53×10^{-2}, fracture stress = 164+Y; for square, radius = 2.57×10^{-4} inches, fracture stress = 278+Y; for diamond, radius = 2.07×10^{-3} inches, fracture stress = 143+Y. (DR = dimpled rupture, CF = cleavage fracture, GBF = grain-boundary fracture.)

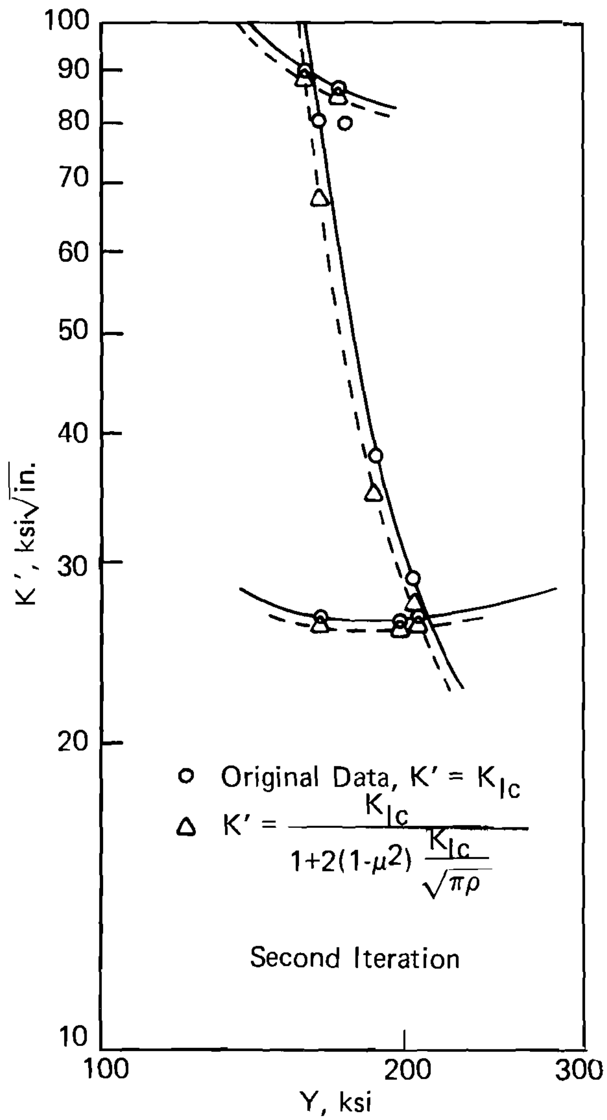


Figure 7. Overlay matches for Figure 6.

Thus, using the $\sqrt{\pi\rho} = 0.0272$ value for the $\sqrt{\pi\rho}$ term in the denominator of K', we proceed to plot log K' versus log Y and, by matching the f overlay to this plot, to determine new and better values for f and ρ. Here

$$K' = \frac{K_{Ic}}{1 + \frac{2(1-\mu^2)\ K_{Ic}}{E\sqrt{\pi\rho}}} = \frac{K_{Ic}}{1 + \frac{0.00006583\ K_{Ic}}{\sqrt{\pi\rho}}}$$

$$= \frac{K_{Ic}}{1 + \frac{0.00006583\ K_{Ic}}{0.0272}} = \frac{K_{Ic}}{1 + 0.00242\ K_{Ic}}$$

with $\mu = 0.28$ and E = 28,000 ksi. Hence we have

Tempering Temperature deg F	Y	K_{Ic}	f/Y
600	163.5	67.4	1.700
800	186.5	34.8	1.491
900	203.5	27.1	1.366

$f = 278$, $f\sqrt{\pi\rho} = 8.08$, $\sqrt{\pi\rho} = 0.0291$

For our third iteration

$$K' = \frac{K_{Ic}}{1 + \frac{0.00006583\ K_{Ic}}{0.0291}} = \frac{K_{Ic}}{1 + 0.00226\ K_{Ic}}$$

and hence

Tempering Temperature deg F	Y	K_{Ic}	f/Y
600	163.5	68.1	1.700
800	186.5	35	1.491
900	203.5	27.2	1.366

$f = 278$, $f\sqrt{\pi\rho} = 7.9$, $\sqrt{\pi\rho} = 0.0284$, $\rho = 2.57 \times 10^{-4}$ in.

Obviously this third iteration was not warranted by the precision of our data or graphs.

Proceeding in the same way for tempers of 1000 F, 1100 F, and 1200 F, we find that f = 142.5 ksi, $f\sqrt{\pi\rho} = 11.5$, and $\rho = 0.00207$ in.

It is useful to note that when, as here, the K_{Ic} values differ little from one another while the Y values vary markedly, the match must lie in the vicinity of the minimum of the overlay which is at 2.26, 1.27, approximately. That is,

$$\frac{K_{Ic}}{f\sqrt{\pi\rho}} = 2.26 \text{ and } \frac{Y}{f} = 1.27$$

where K_{Ic} and Y are at the minimum of the overlay. Thus, since the K_{Ic} values are almost alike we can determine $f\sqrt{\pi\rho}$ quite accurately and simply. In this case,

$$f\sqrt{\pi\rho} = \frac{26.3}{2.26} = 11.6.$$

On the other hand, it is hard to know precisely what the value of Y at the minimum is, i.e., Y exactly where K_{Ic} is minimum, considering the usual scatter in K_{Ic} data and thus the value of f is correspondingly vague. In the present case we estimate the required value of Y by taking it to be midway between the extreme values of Y, i.e.,

$$f = \frac{185}{1.27} = 145 \text{ ksi}.$$

Now, considering the lowest tempering temperatures, we see that the 400 F and 500 F data are not correlated, i.e., matched, by the overlay when it is positioned to cover the 600 F, 800 F, and 900 F points first treated above.

These two data points are matched by

$f = 164$ ksi, $f\sqrt{\pi\rho} = 36$, $\rho = 0.0153$ in.

However, since we have matched only two points instead of three we have no assurance that our results are valid except that they correspond to a curve that is very close to that which the authors of Figure 6 have put through these two points.

Recapitulating, we see from Figure 7, which shows the fit of our overlays to the three sets of data which we have matched and thus provides points in, between, and beyond the data, that there are three distinct regimes governed by their distinctive f and ρ and here named by a fracture appearance characteristic, as follows.

Tempering Temperature, deg F	f, ksi	ρ, in.	Name
400 to 500+	164	0.0153	DR
500+ to 900+	278	0.000257	CF
900+ to 1200	142.5	0.00207	GBF

In giving the names DR, dimpled rupture, CF, cleavage fracture, and GBF, grain boundary fracture, to these regimes we tentatively surmise that under ideal conditions we cannot have two f and two ρ values at the same tempering temperature, that one fracture mechanism is controlling. Thus between tempers of 400 and 500+ F, DR is controlling; between 500+ F and 900+ F, CF is controlling, no matter how small an amount of CF there may be; and between 900+ F and 1200 F, GBF is controlling, no matter how small an amount of GBF there may be. In this we assume that the appearance of CF must have started just above a temper of 500 F and the appearance of GBF must have started just above a temper of 900 F.

It will be noted that the values of ρ seem intuitively consistent, since CF would seem to be associated with a very sharp crack, GBF with a fairly sharp crack, and DR with a dull crack, one whose radius might be relatively easy to detect.

Example 4: Use of Tables for L Versus $\nmid$, $\nmid$ Versus L

In the examples that follow we consider the case where $f = F - Y$ and ρ are independent of Y. Also we assume, as is typically the case, that the small $2(1-\mu^2)\ Y/E$ term may be neglected.

To begin with, we note a property of the L versus $\nmid$ relationship. Since this is

$$\frac{Y\sqrt{\pi\rho}}{K} = \text{function of } \nmid$$

where $\nmid = f/Y = (F/Y)-1,$

that $$K = \frac{Y\sqrt{\pi\rho}}{\text{function of } \nmid}$$

and, since $G = K^2\ (1-\mu^2)/E$,

$$G = \frac{(1-\mu^2)(Y^2/E)\rho\pi}{(\text{function of } \nmid)^2}.$$

Thus K is directly proportional to $\sqrt{\rho}$ and G is directly proportional to ρ for any yield strength, so long as the fracture strength is not changed by the means employed to change ρ. This relationship epitomizes the metallurgical importance of ρ. Occasionally simple inspection of experimental data will demonstrate the constancy of ρ. This suggests the existence of a parameter such as ρ in the make-up of K. That is, the heights K_{Ic} or G of curves characterized by a parameter P, e.g., sulfur content, are proportional to one another and hence ρ = function of P. It is reminiscent of the role of absolute temperature, an integrating factor, in, for example, curves of pv = constant = nRT. See Figure 8.

The quantitative effect of fracture strength on toughness is not so easily summarized, or visualized, being buried in the function of $\nmid$.

We now use the formula for K in a case where we presume f and ρ do not depend on Y, namely where the sulfur content of steel is varied. We find[1]

$$10^{-4}\rho = 3 + 8.3\ \exp[-35\ Su]$$

where ρ is in inches and sulfur Su is in percent.

Now, if we take $K_{Ic} = 55$ ksi$\sqrt{\text{in.}}$ at a 700 F temper with Su = 0.01%, what toughness can we expect if the sulfur level is raised to

Su = 0.02% ? — to Su = 0.03%?

We have from the above formulae,

$$\frac{K_{Ic},\ Su = 0.02}{55} = \frac{3 + 8.3\ \exp[(-35)(0.02)]}{3 + 8.3\ \exp[(-35)(0.01)]}$$

i.e.,

K_{Ic}, Su = 0.02%, is 49.34

and similarly

K_{Ic}, Su = 0.03%, is 44.93.

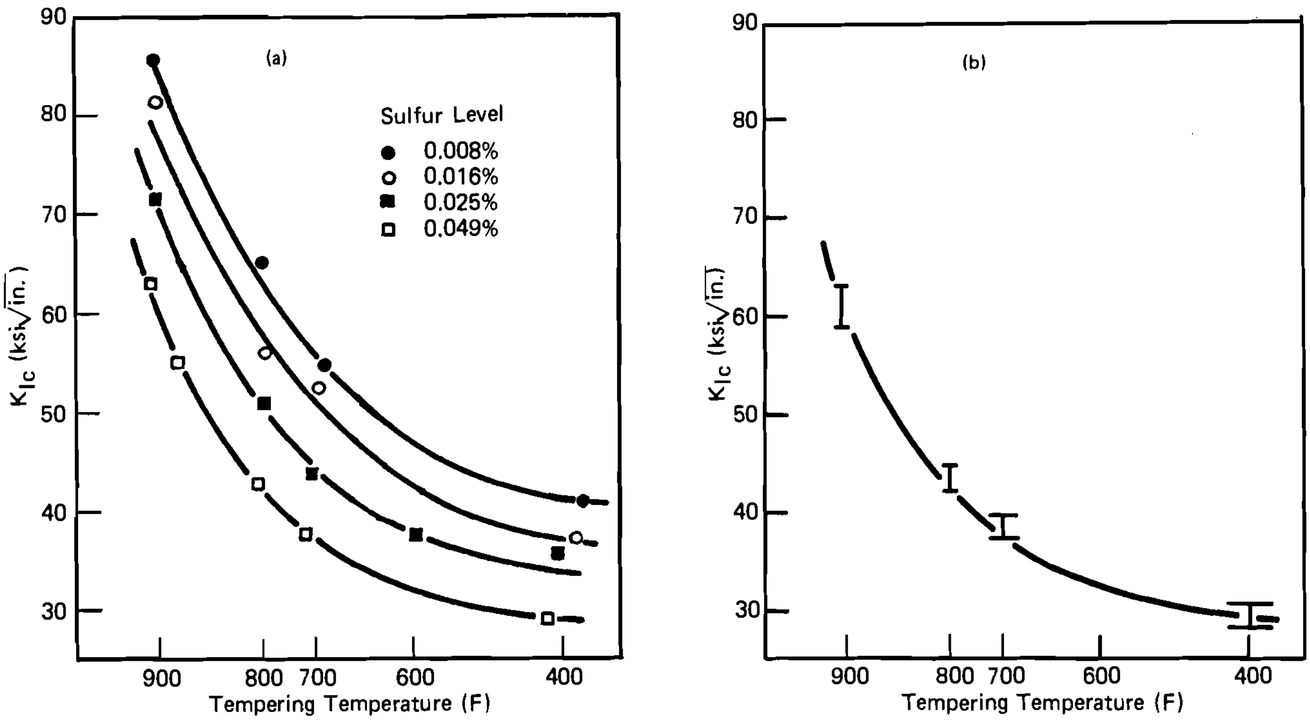


Figure 8. Relationship between toughness curves of 0.45 C-Ni-Cr-Mo steels showing that the form of K_{Ic} versus tempering temperature is independent of sulfur level. (a) Plot and data of Birkle, Wei, and Pellisier, Ref. 8. (b) Reproduction of 0.044% curve of (a) showing range of all data when K_{Ic} data of 0.008% curve is divided by 1.46, data of 0.016% curve is divided by 1.33, and data of 0.025% curve is divided by 1.19.

Toughness generally increases rapidly with increase in tempering temperature through its effect on yield strength and thus the above effect of increase in sulfur level would be substantially greater at higher tempering temperature. Hence, this calculation shows how scatter in K_{Ic} determinations may be expected in the absence of careful control of sulfur and emphasizes the importance of estimates made from theory.

Still referring to the base material in the above example, we select a case involving use of the tables. We suppose an initial carbon content $C_1 = 0.45\%$ and an initial yield strength of 224 ksi at room temperature corresponding to the 700 F temper. We had a crack tip radius

$$\rho = 3 + 8.3 \exp[(-35)(0.01)] \times 10^{-4}$$
$$= 8.849 \times 10^{-4} \text{ in.}$$

We ask, what is the fracture parameter f = F - Y? We have

$$L = \frac{22\sqrt{\pi(8.849 \times 10^{-4})}}{55} = 0.2147.$$

From the L versus ł table

ł = 1.360.

8. BIRKLE, A. J., WEI, R. P., and PELLISIER, G. E. *Analysis of Plane Strain Fracture in a Series of 0.45C-Ni-Cr-Mo Steel with Different Sulfur Contents.* ASM Trans. Quar. 59, no. 4, December 1966, p. 981.

Therefore

$$f = \mathfrak{f}Y = (1.360)(224) = 304.6 \text{ ksi}.$$

Now we presume that if we change the carbon content from C_1 to C_2 that f and ρ will not change and that Y will become (from author's research)

$$Y_2 = Y_1 + 250\ (\sqrt{C_2} - \sqrt{C_1}),$$

comparisons being made at the same test temperature.

We ask, what is the effect on K_{Ic} of lowering the carbon content to $C = C_2 = 0.35\%$?

The yield strength becomes

$$Y_2 = 224 + 250\ (\sqrt{0.35} - \sqrt{0.45}) = 204.2 \text{ ksi}.$$

Thus

$$\mathfrak{f} = \frac{f}{Y} = \frac{304.6}{204.2} = 1.4995$$

and, from the $\mathfrak{f}$ versus L table,

$$L = 0.1388.$$

Therefore,

$$L = 0.1388 = \frac{204.2\sqrt{\pi(8.8489\times10^{-4})}}{K_{Ic}}$$

or

$$K_{Ic} = 77.57 \text{ ksi}\sqrt{\text{in.}}$$

Again, the change that can be wrought by metallurgical control is evident, as is the importance of the L versus $\mathfrak{f}$ relationship.

Naturally, the overlay method could be substituted for some of the computations above, but the tables are accurate, handy, good for single computations, special cases, and to make overlays to cover special regions. They are independent of special assumptions as to how fracture stress varies with yield strength and thus may be used to construct overlays corresponding to such assumptions.

Example 5: Machined-In Notches in Lieu of Cracks; Relation of Notch Stress Concentration to K_{Ic}; Determination of Fracture Stress Law

In previous examples we have used actual crack data to find both fracture stress F and effective crack tip radius ρ from plots of K_{Ic} versus yield strength Y, for cases such that it was possible to match the plots and overlays provided in this report.

Here, however, we discuss cases where a narrow notch of selected depth "a" and tip radius ρ has been machined into a test specimen in order to determine F independently of ρ and "a" and the fracture stress laws assumed in constructing

the overlays. Thus, for example, by testing specimens of a material heat treated to different yield strengths,* we may find the actual dependence of F on Y and from this construct overlays corresponding to this fracture law, if this is desired.

Also we may assume the law relating F to Y is rectilinear which, of course, is permissible for reasonable variations in Y (a variation containing a jump in Y associated with a phase change would be unreasonable!) and determine the constants of linearity, which we denote by β and f, through utilization of either the formulae in Appendix A or the β overlays included in this paper.

The present analysis is based on formulae limited to small-scale (measured in tip radii) plasticity, thus on test conditions similar to those under which K_{Ic} determinations are made.

The analysis utilizes the stress concentration factors of narrow notches, independently of whether these are determined analytically or experimentally (e.g., photoelastically), to determine the stress intensity factor K. It is based on the assumption that failure occurs when a stress equal to the nil ductility fracture stress is reached, no distinction being made, for uniform material, between the stress causing failure in the presence of cracks and in the presence of narrow notches. Obviously, on this concept, superposition of a hydrostatic pressure on the loading stress would reduce the maximum stress whose equality with the fracture stress would mean failure. One may conclude that this reduction in potential for failure is lost, or not concurred in, in treating failure by the usual toughness criterion of $K = K_{Ic}$ or $G = G_{Ic}$ for, in devising the usual formulae used for K by the stress singularity methods, a crack under no internal pressure is assumed and the energy release rate G, from which K is customarily given by the formula $K^2 = EG/(1-\mu^2)$, is independent of the hydrostatic pressure.

When failure occurs, using the machined-in narrow notch method, an initial cohesive failure below the notch surface, followed by gross separation, is to be expected. Our analysis is limited to this initial failure and its occurrence must be noted by (1) the load at which it occurs and/or (2) the distance of the initial notch failure below the notch tip.

The occurrence of the initial failure and hence failure load may be detected by a change in the general shape of the loading curve, or approximated by use of definitions such as those used to determine critical load in standard determinations of K_{Ic} or observed acoustically. Occasionally, it may be assumed to be simultaneous with lateral surface breakthrough.

The position of the initial failure is ordinarily determined by inspection of the completely broken specimen.

With transparent materials the occurrence and position of the initial subsurface fracture may sometimes be observed directly.[2]

The analytical expression for stress intensity factor K is the coefficient of an $r^{-\frac{1}{2}}$ power singularity in the elastic stress analysis for specimens with a crack,

*As pointed out in Example 4, K and hence F may vary with other variables, especially sulfur, at constant Y, so that the fracture law may not be simply F = f (Y).

where r is the radial distance from the crack tip. However, our analysis here is based on our observation that this coefficient K is* also given by

$$K = (CS/2)\sqrt{\pi a}$$

in terms of the *coefficient of elastic stress concentration for narrow notches* C, which is a function of the geometrical configuration of the loaded object, and the nominal principal loading stress S. Here, on the average the notch and its tip are supposed to lie perpendicularly to the loading stress S.

Thus, if a crack is conceived of as a narrow notch whose boundary is a smooth curve characterized by a tip radius ρ, then no matter how small one assumes this radius to be, the boundary† stress S_{tip} at the tip is practically, for sufficiently large a/ρ,

$$\begin{aligned} S_{tip} = SC\sqrt{a/\rho} &= 2\ K/\sqrt{\pi\rho} \\ &= 2\ K/\{[L - 2(1-\mu^2)Y/E]\,[K/Y]\} \\ &\cong 2\ Y/L \end{aligned}$$

utilizing the above expression for K and $L = Y\sqrt{\pi\rho}/S\sqrt{\pi a}) + 2(1-\mu^2)Y/E$. Note that $S_{tip} = 2\ Y/L$ with K defined either as $K = S\sqrt{\pi a}$ or $K = S\sqrt{a}$ since π may be cancelled out of the expression for L. The use of the approximate expression for L is satisfactory since our machined-in radii are sufficiently large to make the second term in L negligibly small.

The meaning of "sufficiently large a/ρ" in the preceding paragraph is typified by this: In general the expression for maximum elastic notch stress will be given by a more complicated expression than used above, e.g., by $S_{tip} = S(1 + 2\sqrt{a/\rho})$ for an elliptic hole lying across a tension field of loading stress S, but for sufficiently large a/ρ it reduces for practical purposes to $S_{tip} = SC\sqrt{a/\rho}$.

Thus we have connected C, the coefficient of notch stress concentration of any Mode I narrow notch of depth (usually denoted by) "a", and tip radius ρ with K as well as S, Y, and L. S_{tip} may be above the yield strength, but this connection remains valid for our analyses and use of our L versus ɬ curves.

a. Simple Example Using Tables or Formulae, Appendix A

Let a narrow parabolic notch of tip radius $\rho = 0.01''$ be machined into a material of Y = 100 ksi yield strength. Let K, evaluated for the test geometry from the load at initial failure, be 30 ksi $\text{in.}^{\frac{1}{2}}$.

Then $L \cong \dfrac{Y\sqrt{\pi\rho}}{K} = 0.5908$.

From the tables ɬ = 0.7257, thus F = (1 + ɬ) Y = 172.57 ksi.

Using the formulae instead of the tables, we note that L and ɬ are in Region I since in Region I, $2 \geq L \geq 0.5$, and therefore

*It is more natural in the analysis to use the $K = S\sqrt{a}$ definition than the one we use above, $K = S\sqrt{\pi a}$, thus that $K = (CS/2)\sqrt{a}$. In this paper we use the expression containing π because most data and literature conform to it.

†In polar coordinates the boundary near the tip with such a characterization is $r = (\rho/2)\cos^{-2}(\theta/2)$, where θ is measured from the crack (notch) center line.

$\nless = \sqrt{3}/[9/(2-L)-4] = 0.7257$, as before.

To get the distance of the break below the notch tip, we must use one of the formulae of Appendix A since we have not constructed tables from the formulae for distance.

These are, for Region I,

$$\sqrt{3}/\nless = 1/(\overline{Y} - 0.5) + 1/3$$

$$9/(2 - L) = 1/(\overline{Y} - 0.5) + 13/3$$

where L and $\nless$ are to have the values found above.

Either formula gives

$$\overline{Y} - 0.5 = 0.4870$$

which is the distance of the initial break from the nose of the crack, in tip radii. In inches we have $0.4870 \times 0.01 = 0.00487$.

Conversely, if this distance were observed, the formula relating $\nless$ to $\overline{Y}$ could be used to find $\nless$ and the latter to find F.

Obviously, if this distance is regarded as too small to measure, a change of test specimen dimensions would be necessary.

b. Linear F Versus Y Fracture Law

The linear F versus Y fracture law is

$$F = (1 + \beta)\, Y + f$$

where β and f are constants and β is the β of the family of overlays contained herein. It is generally useful; it is not only for piecewise approximation of more general laws. However, care must be taken to see that there are no discontinuities in the region being notched.

With this law, it is apparent that the constants β and f are

$$\beta = \frac{F_1 - F_2}{Y_1 - Y_2} - 1 = \frac{\nless_1 Y_1 - \nless_2 Y_2}{Y_1 - Y_2}$$

$$f = -\frac{F_1 Y_2 - F_2 Y_1}{Y_1 - Y_2} = -\frac{Y_1 Y_2 (\nless_1 - \nless_2)}{Y_1 - Y_2}$$

and

(1) For $\beta = -1$, $F = f =$ constant and $F_1 = F_2$.
The fracture stress is constant.

(2) For $\beta = 0$, $F = Y + f$ and $F_1 - F_2 = Y_1 - Y_2$.
The change in fracture strength equals the change in Y.

(3) If $f = 0$, $\beta = \nless = \nless_1 = \nless_2 = F_1/Y_1 - 1 = F_2/Y_2 - 1$; $F_2 = F_1 Y_2/Y_1$.

(4) If $Y_1 = Y_2$, $F_1 = F_2 = F$.

If this condition holds, there will be no difference in F even though the specimens have different tip radii. If one wants the effect of Y on F, one must vary Y, e.g., one cannot simply vary tip radii to find the effect of Y on f.

Let us assume that we have made another experiment with another specimen in addition to that of Example 5a, using a yield strength of 150 ksi and the same tip radius of 0.01 inch and find that K computed from the load at initial failure is 20 ksi in.$^{\frac{1}{2}}$.

Then $L = 1.3293$ (Region I)
$\dagger = 0.1839$
$F = (1.1839)(150) = 177.59$ ksi
$\overline{Y} - 0.5 = 0.1101$, and
$(0.1101)(0.01) = 0.00110$ inch.

Thus from the results above and of Example 5a we find that

$\beta = -0.8996$
$f = 162.53$ ksi, so that
$F = 0.1004Y + 162.53$.

In our illustration here we have used arbitrarily assumed data. But if there were a material corresponding to these figures, our $\beta = -0.9$ overlay should be used for its graphical analysis.

We have assumed that K was determined from A.S.T.M. specification type procedures. However, in determining K_{Ic} we have assumed that the load corresponding to initial failure is detectable and used if different from that specified in the standards.

Instead of doing that, we might have assumed that the distance of the initial fracture from the nose was observed on the fractured surface of the specimen to be given by the values of $\overline{Y} - 0.5$ computed above, and used the formulae for $\overline{Y} - 0.5$ to compute the L's and $\dagger$'s. From the latter and the yield strength we would then compute the F's and the fracture law.

This latter procedure is far the most appealing. It requires no formula for K and no means of detecting K.

SUMMARY

Along with technical background and proofs, this paper demonstrates the method used by the writers to obtain fracture stress and effective crack tip radius corresponding to the instant of fracture, very simply from log-log plots of K_{Ic} and corresponding yield strength data, using transparent overlays supplied with the paper. It employs examples to illustrate the method, the influence of testing temperature, metallurgical structure, chemistry, and even the existence of the effective crack tip radius. It includes associated tables relating two dimensionless parameters L and $\dagger$, involving toughness, yield strength, crack tip radius, and peak stress at any elastically contained degree of yielding consistent with valid toughness data.

APPENDIX A.

I. Tables of L Versus $\nmid$ and $\nmid$ Versus L (Figure A-1)

$$L = Y\sqrt{\pi\rho}/K + 2(1-\mu^2)Y/E$$

$$\nmid = S_{max}/Y - 1$$

where

Y = yield strength
E = Young's modulus of elasticity
K = toughness
ρ = crack tip radius
μ = Poisson's ratio
$\nmid$ = maximum shear trajectory angle change in yielded region
S_{max} = maximum stress (can be a fracture stress)

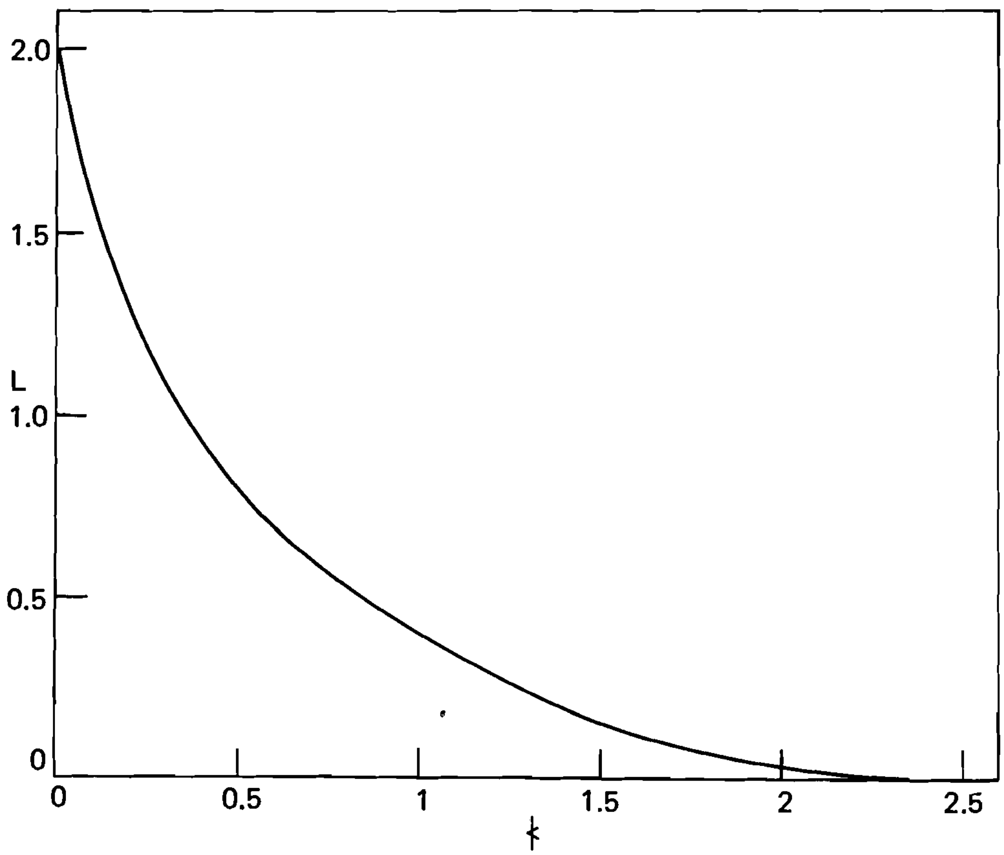


Figure A-1. L versus $\nmid$.

TABLES OF L VERSUS ƚ

L	ƚ	L	ƚ	L	ƚ	L	ƚ
2.0000	.0000	1.9500	.0098	1.9000	.0201	1.8500	.0309
1.9990	.0002	1.9490	.0100	1.8990	.0204	1.8490	.0312
1.9980	.0004	1.9480	.0102	1.8980	.0206	1.8480	.0314
1.9970	.0006	1.9470	.0104	1.8970	.0208	1.8470	.0316
1.9960	.0008	1.9460	.0106	1.8960	.0210	1.8460	.0318
1.9950	.0010	1.9450	.0108	1.8950	.0212	1.8450	.0320
1.9940	.0012	1.9440	.0111	1.8940	.0214	1.8440	.0323
1.9930	.0014	1.9430	.0113	1.8930	.0216	1.8430	.0325
1.9920	.0015	1.9420	.0115	1.8920	.0218	1.8420	.0327
1.9910	.0017	1.9410	.0117	1.8910	.0220	1.8410	.0329
1.9900	.0019	1.9400	.0119	1.8900	.0223	1.8400	.0331
1.9890	.0021	1.9390	.0121	1.8890	.0225	1.8390	.0334
1.9880	.0023	1.9380	.0123	1.8880	.0227	1.8380	.0336
1.9870	.0025	1.9370	.0125	1.8870	.0229	1.8370	.0338
1.9860	.0027	1.9360	.0127	1.8860	.0231	1.8360	.0340
1.9850	.0029	1.9350	.0129	1.8850	.0233	1.8350	.0343
1.9840	.0031	1.9340	.0131	1.8840	.0235	1.8340	.0345
1.9830	.0033	1.9330	.0133	1.8830	.0238	1.8330	.0347
1.9820	.0035	1.9320	.0135	1.8820	.0240	1.8320	.0349
1.9810	.0037	1.9310	.0137	1.8810	.0242	1.8310	.0352
1.9800	.0039	1.9300	.0139	1.8800	.0244	1.8300	.0354
1.9790	.0041	1.9290	.0141	1.8790	.0246	1.8290	.0356
1.9780	.0043	1.9280	.0143	1.8780	.0248	1.8280	.0358
1.9770	.0045	1.9270	.0145	1.8770	.0250	1.8270	.0361
1.9760	.0047	1.9260	.0147	1.8760	.0253	1.8260	.0363
1.9750	.0049	1.9250	.0149	1.8750	.0255	1.8250	.0365
1.9740	.0051	1.9240	.0151	1.8740	.0257	1.8240	.0367
1.9730	.0053	1.9230	.0153	1.8730	.0259	1.8230	.0370
1.9720	.0055	1.9220	.0155	1.8720	.0261	1.8220	.0372
1.9710	.0057	1.9210	.0158	1.8710	.0263	1.8210	.0374
1.9700	.0059	1.9200	.0160	1.8700	.0266	1.8200	.0377
1.9690	.0060	1.9190	.0162	1.8690	.0268	1.8190	.0379
1.9680	.0062	1.9180	.0164	1.8680	.0270	1.8180	.0381
1.9670	.0064	1.9170	.0166	1.8670	.0272	1.8170	.0383
1.9660	.0066	1.9160	.0168	1.8660	.0274	1.8160	.0386
1.9650	.0068	1.9150	.0170	1.8650	.0276	1.8150	.0388
1.9640	.0070	1.9140	.0172	1.8640	.0279	1.8140	.0390
1.9630	.0072	1.9130	.0174	1.8630	.0281	1.8130	.0392
1.9620	.0074	1.9120	.0176	1.8620	.0283	1.8120	.0395
1.9610	.0076	1.9110	.0178	1.8610	.0285	1.8110	.0397
1.9600	.0078	1.9100	.0180	1.8600	.0287	1.8100	.0399
1.9590	.0080	1.9090	.0183	1.8590	.0289	1.8090	.0402
1.9580	.0082	1.9080	.0185	1.8580	.0292	1.8080	.0404
1.9570	.0084	1.9070	.0187	1.8570	.0294	1.8070	.0406
1.9560	.0086	1.9060	.0189	1.8560	.0296	1.8060	.0409
1.9550	.0088	1.9050	.0191	1.8550	.0298	1.8050	.0411
1.9540	.0090	1.9040	.0193	1.8540	.0300	1.8040	.0413
1.9530	.0092	1.9030	.0195	1.8530	.0303	1.8030	.0416
1.9520	.0094	1.9020	.0197	1.8520	.0305	1.8020	.0418
1.9510	.0096	1.9010	.0199	1.8510	.0307	1.8010	.0420
						1.8000	.0422

L	ƚ	L	ƚ	L	ƚ	L	ƚ
1.8000	.0422	1.7500	.0541	1.7000	.0666	1.6500	.0798
1.7990	.0425	1.7490	.0544	1.6990	.0669	1.6490	.0800
1.7980	.0427	1.7480	.0546	1.6980	.0671	1.6480	.0803
1.7970	.0429	1.7470	.0549	1.6970	.0674	1.6470	.0806
1.7960	.0432	1.7460	.0551	1.6960	.0676	1.6460	.0808
1.7950	.0434	1.7450	.0553	1.6950	.0679	1.6450	.0811
1.7940	.0436	1.7440	.0556	1.6940	.0682	1.6440	.0814
1.7930	.0439	1.7430	.0558	1.6930	.0684	1.6430	.0817
1.7920	.0441	1.7420	.0561	1.6920	.0687	1.6420	.0819
1.7910	.0443	1.7410	.0563	1.6910	.0689	1.6410	.0822
1.7900	.0446	1.7400	.0566	1.6900	.0692	1.6400	.0825
1.7890	.0448	1.7390	.0568	1.6890	.0695	1.6390	.0828
1.7880	.0450	1.7380	.0571	1.6880	.0697	1.6380	.0830
1.7870	.0453	1.7370	.0573	1.6870	.0700	1.6370	.0833
1.7860	.0455	1.7360	.0576	1.6860	.0702	1.6360	.0836
1.7850	.0457	1.7350	.0578	1.6850	.0705	1.6350	.0838
1.7840	.0460	1.7340	.0581	1.6840	.0707	1.6340	.0841
1.7830	.0462	1.7330	.0583	1.6830	.0710	1.6330	.0844
1.7820	.0465	1.7320	.0585	1.6820	.0713	1.6320	.0847
1.7810	.0467	1.7310	.0588	1.6810	.0715	1.6310	.0849
1.7800	.0469	1.7300	.0590	1.6800	.0718	1.6300	.0852
1.7790	.0472	1.7290	.0593	1.6790	.0721	1.6290	.0855
1.7780	.0474	1.7280	.0595	1.6780	.0723	1.6280	.0858
1.7770	.0476	1.7270	.0598	1.6770	.0726	1.6270	.0860
1.7760	.0479	1.7260	.0600	1.6760	.0728	1.6260	.0863
1.7750	.0481	1.7250	.0603	1.6750	.0731	1.6250	.0866
1.7740	.0483	1.7240	.0605	1.6740	.0734	1.6240	.0869
1.7730	.0486	1.7230	.0608	1.6730	.0736	1.6230	.0872
1.7720	.0488	1.7220	.0610	1.6720	.0739	1.6220	.0874
1.7710	.0491	1.7210	.0613	1.6710	.0742	1.6210	.0877
1.7700	.0493	1.7200	.0615	1.6700	.0744	1.6200	.0880
1.7690	.0495	1.7190	.0618	1.6690	.0747	1.6190	.0883
1.7680	.0498	1.7180	.0620	1.6680	.0750	1.6180	.0885
1.7670	.0500	1.7170	.0623	1.6670	.0752	1.6170	.0888
1.7660	.0503	1.7160	.0626	1.6660	.0755	1.6160	.0891
1.7650	.0505	1.7150	.0628	1.6650	.0757	1.6150	.0894
1.7640	.0507	1.7140	.0631	1.6640	.0760	1.6140	.0897
1.7630	.0510	1.7130	.0633	1.6630	.0763	1.6130	.0899
1.7620	.0512	1.7120	.0636	1.6620	.0765	1.6120	.0902
1.7610	.0515	1.7110	.0638	1.6610	.0768	1.6110	.0905
1.7600	.0517	1.7100	.0641	1.6600	.0771	1.6100	.0908
1.7590	.0519	1.7090	.0643	1.6590	.0773	1.6090	.0911
1.7580	.0522	1.7080	.0646	1.6580	.0776	1.6080	.0914
1.7570	.0524	1.7070	.0648	1.6570	.0779	1.6070	.0916
1.7560	.0527	1.7060	.0651	1.6560	.0782	1.6060	.0919
1.7550	.0529	1.7050	.0653	1.6550	.0784	1.6050	.0922
1.7540	.0532	1.7040	.0656	1.6540	.0787	1.6040	.0925
1.7530	.0534	1.7030	.0658	1.6530	.0790	1.6030	.0928
1.7520	.0536	1.7020	.0661	1.6520	.0792	1.6020	.0931
1.7510	.0539	1.7010	.0664	1.6510	.0795	1.6010	.0933
						1.6000	.0936

L	k	L	k	L	k	L	k
1.6000	.0936	1.5500	.1083	1.5000	.1237	1.4500	.1401
1.5990	.0939	1.5490	.1086	1.4990	.1240	1.4490	.1404
1.5980	.0942	1.5480	.1089	1.4980	.1244	1.4480	.1408
1.5970	.0945	1.5470	.1092	1.4970	.1247	1.4470	.1411
1.5960	.0948	1.5460	.1095	1.4960	.1250	1.4460	.1414
1.5950	.0951	1.5450	.1098	1.4950	.1253	1.4450	.1418
1.5940	.0953	1.5440	.1101	1.4940	.1256	1.4440	.1421
1.5930	.0956	1.5430	.1104	1.4930	.1260	1.4430	.1425
1.5920	.0959	1.5420	.1107	1.4920	.1263	1.4420	.1428
1.5910	.0962	1.5410	.1110	1.4910	.1266	1.4410	.1431
1.5900	.0965	1.5400	.1113	1.4900	.1269	1.4400	.1435
1.5890	.0968	1.5390	.1116	1.4890	.1272	1.4390	.1438
1.5880	.0971	1.5380	.1119	1.4880	.1276	1.4380	.1442
1.5870	.0973	1.5370	.1122	1.4870	.1279	1.4370	.1445
1.5860	.0976	1.5360	.1125	1.4860	.1282	1.4360	.1448
1.5850	.0979	1.5350	.1128	1.4850	.1285	1.4350	.1452
1.5840	.0982	1.5340	.1131	1.4840	.1289	1.4340	.1455
1.5830	.0985	1.5330	.1134	1.4830	.1292	1.4330	.1459
1.5820	.0988	1.5320	.1137	1.4820	.1295	1.4320	.1462
1.5810	.0991	1.5310	.1140	1.4810	.1298	1.4310	.1466
1.5800	.0994	1.5300	.1143	1.4800	.1302	1.4300	.1469
1.5790	.0997	1.5290	.1146	1.4790	.1305	1.4290	.1473
1.5780	.1000	1.5280	.1149	1.4780	.1308	1.4280	.1476
1.5770	.1003	1.5270	.1153	1.4770	.1311	1.4270	.1479
1.5760	.1005	1.5260	.1156	1.4760	.1315	1.4260	.1483
1.5750	.1008	1.5250	.1159	1.4750	.1318	1.4250	.1486
1.5740	.1011	1.5240	.1162	1.4740	.1321	1.4240	.1490
1.5730	.1014	1.5230	.1165	1.4730	.1324	1.4230	.1493
1.5720	.1017	1.5220	.1168	1.4720	.1328	1.4220	.1497
1.5710	.1020	1.5210	.1171	1.4710	.1331	1.4210	.1500
1.5700	.1023	1.5200	.1174	1.4700	.1334	1.4200	.1504
1.5690	.1026	1.5190	.1177	1.4690	.1338	1.4190	.1507
1.5680	.1029	1.5180	.1180	1.4680	.1341	1.4180	.1511
1.5670	.1032	1.5170	.1184	1.4670	.1344	1.4170	.1514
1.5660	.1035	1.5160	.1187	1.4660	.1347	1.4160	.1518
1.5650	.1038	1.5150	.1190	1.4650	.1351	1.4150	.1521
1.5640	.1041	1.5140	.1193	1.4640	.1354	1.4140	.1525
1.5630	.1044	1.5130	.1196	1.4630	.1357	1.4130	.1528
1.5620	.1047	1.5120	.1199	1.4620	.1361	1.4120	.1532
1.5610	.1050	1.5110	.1202	1.4610	.1364	1.4110	.1535
1.5600	.1053	1.5100	.1206	1.4600	.1367	1.4100	.1539
1.5590	.1056	1.5090	.1209	1.4590	.1371	1.4090	.1543
1.5580	.1059	1.5080	.1212	1.4580	.1374	1.4080	.1546
1.5570	.1062	1.5070	.1215	1.4570	.1377	1.4070	.1550
1.5560	.1065	1.5060	.1218	1.4560	.1381	1.4060	.1553
1.5550	.1068	1.5050	.1221	1.4550	.1384	1.4050	.1557
1.5540	.1071	1.5040	.1224	1.4540	.1387	1.4040	.1560
1.5530	.1074	1.5030	.1228	1.4530	.1391	1.4030	.1564
1.5520	.1077	1.5020	.1231	1.4520	.1394	1.4020	.1567
1.5510	.1080	1.5010	.1234	1.4510	.1398	1.4010	.1571
						1.4000	.1575

L	k	L	k	L	k	L	k
1.4000	.1575	1.3500	.1759	1.3000	.1956	1.2500	.2165
1.3990	.1578	1.3490	.1763	1.2990	.1960	1.2490	.2169
1.3980	.1582	1.3480	.1767	1.2980	.1964	1.2480	.2174
1.3970	.1585	1.3470	.1771	1.2970	.1968	1.2470	.2178
1.3960	.1589	1.3460	.1774	1.2960	.1972	1.2460	.2182
1.3950	.1593	1.3450	.1778	1.2950	.1976	1.2450	.2187
1.3940	.1596	1.3440	.1782	1.2940	.1980	1.2440	.2191
1.3930	.1600	1.3430	.1786	1.2930	.1984	1.2430	.2195
1.3920	.1603	1.3420	.1790	1.2920	.1988	1.2420	.2200
1.3910	.1607	1.3410	.1794	1.2910	.1992	1.2410	.2204
1.3900	.1611	1.3400	.1797	1.2900	.1996	1.2400	.2209
1.3890	.1614	1.3390	.1801	1.2890	.2000	1.2390	.2213
1.3880	.1618	1.3380	.1805	1.2880	.2005	1.2380	.2217
1.3870	.1621	1.3370	.1809	1.2870	.2009	1.2370	.2222
1.3860	.1625	1.3360	.1813	1.2860	.2013	1.2360	.2226
1.3850	.1629	1.3350	.1817	1.2850	.2017	1.2350	.2231
1.3840	.1632	1.3340	.1821	1.2840	.2021	1.2340	.2235
1.3830	.1636	1.3330	.1824	1.2830	.2025	1.2330	.2239
1.3820	.1640	1.3320	.1828	1.2820	.2029	1.2320	.2244
1.3810	.1643	1.3310	.1832	1.2810	.2034	1.2310	.2248
1.3800	.1647	1.3300	.1836	1.2800	.2038	1.2300	.2253
1.3790	.1651	1.3290	.1840	1.2790	.2042	1.2290	.2257
1.3780	.1654	1.3280	.1844	1.2780	.2046	1.2280	.2262
1.3770	.1658	1.3270	.1848	1.2770	.2050	1.2270	.2266
1.3760	.1662	1.3260	.1852	1.2760	.2054	1.2260	.2271
1.3750	.1665	1.3250	.1856	1.2750	.2059	1.2250	.2275
1.3740	.1669	1.3240	.1860	1.2740	.2063	1.2240	.2280
1.3730	.1673	1.3230	.1864	1.2730	.2067	1.2230	.2284
1.3720	.1676	1.3220	.1868	1.2720	.2071	1.2220	.2289
1.3710	.1680	1.3210	.1871	1.2710	.2075	1.2210	.2293
1.3700	.1684	1.3200	.1875	1.2700	.2080	1.2200	.2298
1.3690	.1688	1.3190	.1879	1.2690	.2084	1.2190	.2302
1.3680	.1691	1.3180	.1883	1.2680	.2088	1.2180	.2307
1.3670	.1695	1.3170	.1887	1.2670	.2092	1.2170	.2311
1.3660	.1699	1.3160	.1891	1.2660	.2096	1.2160	.2316
1.3650	.1703	1.3150	.1895	1.2650	.2101	1.2150	.2320
1.3640	.1706	1.3140	.1899	1.2640	.2105	1.2140	.2325
1.3630	.1710	1.3130	.1903	1.2630	.2109	1.2130	.2329
1.3620	.1714	1.3120	.1907	1.2620	.2113	1.2120	.2334
1.3610	.1718	1.3110	.1911	1.2610	.2118	1.2110	.2338
1.3600	.1721	1.3100	.1915	1.2600	.2122	1.2100	.2343
1.3590	.1725	1.3090	.1919	1.2590	.2126	1.2090	.2348
1.3580	.1729	1.3080	.1923	1.2580	.2131	1.2080	.2352
1.3570	.1733	1.3070	.1927	1.2570	.2135	1.2070	.2357
1.3560	.1736	1.3060	.1931	1.2560	.2139	1.2060	.2361
1.3550	.1740	1.3050	.1935	1.2550	.2143	1.2050	.2366
1.3540	.1744	1.3040	.1939	1.2540	.2148	1.2040	.2371
1.3530	.1748	1.3030	.1943	1.2530	.2152	1.2030	.2375
1.3520	.1751	1.3020	.1947	1.2520	.2156	1.2020	.2380
1.3510	.1755	1.3010	.1951	1.2510	.2161	1.2010	.2384
						1.2000	.2389

L	ꝁ	L	ꝁ	L	ꝁ	L	ꝁ
1.2000	.2389	1.1500	.2629	1.1000	.2887	1.0500	.3164
1.1990	.2394	1.1490	.2634	1.0990	.2892	1.0490	.3170
1.1980	.2398	1.1480	.2639	1.0980	.2897	1.0480	.3176
1.1970	.2403	1.1470	.2644	1.0970	.2903	1.0470	.3182
1.1960	.2408	1.1460	.2649	1.0960	.2908	1.0460	.3187
1.1950	.2412	1.1450	.2654	1.0950	.2914	1.0450	.3193
1.1940	.2417	1.1440	.2659	1.0940	.2919	1.0440	.3199
1.1930	.2422	1.1430	.2664	1.0930	.2924	1.0430	.3205
1.1920	.2426	1.1420	.2669	1.0920	.2930	1.0420	.3211
1.1910	.2431	1.1410	.2674	1.0910	.2935	1.0410	.3216
1.1900	.2436	1.1400	.2679	1.0900	.2941	1.0400	.3222
1.1890	.2440	1.1390	.2684	1.0890	.2946	1.0390	.3228
1.1880	.2445	1.1380	.2689	1.0880	.2951	1.0380	.3234
1.1870	.2450	1.1370	.2694	1.0870	.2957	1.0370	.3240
1.1860	.2454	1.1360	.2699	1.0860	.2962	1.0360	.3246
1.1850	.2459	1.1350	.2704	1.0850	.2968	1.0350	.3252
1.1840	.2464	1.1340	.2709	1.0840	.2973	1.0340	.3258
1.1830	.2469	1.1330	.2714	1.0830	.2979	1.0330	.3264
1.1820	.2473	1.1320	.2720	1.0820	.2984	1.0320	.3269
1.1810	.2478	1.1310	.2725	1.0810	.2990	1.0310	.3275
1.1800	.2483	1.1300	.2730	1.0800	.2995	1.0300	.3281
1.1790	.2488	1.1290	.2735	1.0790	.3001	1.0290	.3287
1.1780	.2493	1.1280	.2740	1.0780	.3006	1.0280	.3293
1.1770	.2497	1.1270	.2745	1.0770	.3012	1.0270	.3299
1.1760	.2502	1.1260	.2750	1.0760	.3017	1.0260	.3305
1.1750	.2507	1.1250	.2755	1.0750	.3023	1.0250	.3311
1.1740	.2512	1.1240	.2761	1.0740	.3028	1.0240	.3317
1.1730	.2516	1.1230	.2766	1.0730	.3034	1.0230	.3323
1.1720	.2521	1.1220	.2771	1.0720	.3040	1.0220	.3329
1.1710	.2526	1.1210	.2776	1.0710	.3045	1.0210	.3335
1.1700	.2531	1.1200	.2781	1.0700	.3051	1.0200	.3341
1.1690	.2536	1.1190	.2787	1.0690	.3056	1.0190	.3347
1.1680	.2541	1.1180	.2792	1.0680	.3062	1.0180	.3353
1.1670	.2545	1.1170	.2797	1.0670	.3068	1.0170	.3359
1.1660	.2550	1.1160	.2802	1.0660	.3073	1.0160	.3366
1.1650	.2555	1.1150	.2807	1.0650	.3079	1.0150	.3372
1.1640	.2560	1.1140	.2813	1.0640	.3084	1.0140	.3378
1.1630	.2565	1.1130	.2818	1.0630	.3090	1.0130	.3384
1.1620	.2570	1.1120	.2823	1.0620	.3096	1.0120	.3390
1.1610	.2575	1.1110	.2828	1.0610	.3101	1.0110	.3396
1.1600	.2580	1.1100	.2834	1.0600	.3107	1.0100	.3402
1.1590	.2585	1.1090	.2839	1.0590	.3113	1.0090	.3408
1.1580	.2589	1.1080	.2844	1.0580	.3118	1.0080	.3414
1.1570	.2594	1.1070	.2849	1.0570	.3124	1.0070	.3421
1.1560	.2599	1.1060	.2855	1.0560	.3130	1.0060	.3427
1.1550	.2604	1.1050	.2860	1.0550	.3136	1.0050	.3433
1.1540	.2609	1.1040	.2865	1.0540	.3141	1.0040	.3439
1.1530	.2614	1.1030	.2871	1.0530	.3147	1.0030	.3445
1.1520	.2619	1.1020	.2876	1.0520	.3153	1.0020	.3452
1.1510	.2624	1.1010	.2881	1.0510	.3158	1.0010	.3458
						1.0000	.3464

L	ƒ	L	ƒ	L	ƒ	L	ƒ
1.0000	.3464	.9500	.3789	.9000	.4142	.8500	.4527
.9990	.3470	.9490	.3796	.8990	.4149	.8490	.4535
.9980	.3477	.9480	.3802	.8980	.4157	.8480	.4543
.9970	.3483	.9470	.3809	.8970	.4164	.8470	.4551
.9960	.3489	.9460	.3816	.8960	.4171	.8460	.4559
.9950	.3495	.9450	.3823	.8950	.4179	.8450	.4567
.9940	.3502	.9440	.3830	.8940	.4186	.8440	.4575
.9930	.3508	.9430	.3836	.8930	.4194	.8430	.4584
.9920	.3514	.9420	.3843	.8920	.4201	.8420	.4592
.9910	.3521	.9410	.3850	.8910	.4209	.8410	.4600
.9900	.3527	.9400	.3857	.8900	.4216	.8400	.4608
.9890	.3533	.9390	.3864	.8890	.4224	.8390	.4616
.9880	.3540	.9380	.3871	.8880	.4231	.8380	.4625
.9870	.3546	.9370	.3878	.8870	.4239	.8370	.4633
.9860	.3552	.9360	.3885	.8860	.4246	.8360	.4641
.9850	.3559	.9350	.3892	.8850	.4254	.8350	.4649
.9840	.3565	.9340	.3898	.8840	.4261	.8340	.4658
.9830	.3571	.9330	.3905	.8830	.4269	.8330	.4666
.9820	.3578	.9320	.3912	.8820	.4276	.8320	.4674
.9810	.3584	.9310	.3919	.8810	.4284	.8310	.4683
.9800	.3591	.9300	.3926	.8800	.4292	.8300	.4691
.9790	.3597	.9290	.3933	.8790	.4299	.8290	.4699
.9780	.3604	.9280	.3940	.8780	.4307	.8280	.4708
.9770	.3610	.9270	.3947	.8770	.4315	.8270	.4716
.9760	.3617	.9260	.3954	.8760	.4322	.8260	.4724
.9750	.3623	.9250	.3962	.8750	.4330	.8250	.4733
.9740	.3630	.9240	.3969	.8740	.4338	.8240	.4741
.9730	.3636	.9230	.3976	.8730	.4345	.8230	.4750
.9720	.3643	.9220	.3983	.8720	.4353	.8220	.4758
.9710	.3649	.9210	.3990	.8710	.4361	.8210	.4767
.9700	.3656	.9200	.3997	.8700	.4369	.8200	.4775
.9690	.3662	.9190	.4004	.8690	.4376	.8190	.4784
.9680	.3669	.9180	.4011	.8680	.4384	.8180	.4792
.9670	.3675	.9170	.4018	.8670	.4392	.8170	.4801
.9660	.3682	.9160	.4026	.8660	.4400	.8160	.4809
.9650	.3689	.9150	.4033	.8650	.4408	.8150	.4818
.9640	.3695	.9140	.4040	.8640	.4416	.8140	.4827
.9630	.3702	.9130	.4047	.8630	.4423	.8130	.4835
.9620	.3708	.9120	.4054	.8620	.4431	.8120	.4844
.9610	.3715	.9110	.4061	.8610	.4439	.8110	.4852
.9600	.3722	.9100	.4069	.8600	.4447	.8100	.4861
.9590	.3728	.9090	.4076	.8590	.4455	.8090	.4870
.9580	.3735	.9080	.4083	.8580	.4463	.8080	.4878
.9570	.3742	.9070	.4091	.8570	.4471	.8070	.4887
.9560	.3748	.9060	.4098	.8560	.4479	.8060	.4896
.9550	.3755	.9050	.4105	.8550	.4487	.8050	.4905
.9540	.3762	.9040	.4112	.8540	.4495	.8040	.4913
.9530	.3769	.9030	.4120	.8530	.4503	.8030	.4922
.9520	.3775	.9020	.4127	.8520	.4511	.8020	.4931
.9510	.3782	.9010	.4134	.8510	.4519	.8010	.4940
						.8000	.4949

L	ꝁ	L	ꝁ	L	ꝁ	L	ꝁ
.8000	.4949	.7500	.5413	.7000	.5925	.6500	.6495
.7990	.4957	.7490	.5422	.6990	.5936	.6490	.6507
.7980	.4966	.7480	.5432	.6980	.5947	.6480	.6519
.7970	.4975	.7470	.5442	.6970	.5958	.6470	.6531
.7960	.4984	.7460	.5452	.6960	.5969	.6460	.6543
.7950	.4993	.7450	.5461	.6950	.5980	.6450	.6555
.7940	.5002	.7440	.5471	.6940	.5990	.6440	.6568
.7930	.5011	.7430	.5481	.6930	.6001	.6430	.6580
.7920	.5020	.7420	.5491	.6920	.6012	.6420	.6592
.7910	.5029	.7410	.5501	.6910	.6023	.6410	.6604
.7900	.5038	.7400	.5511	.6900	.6034	.6400	.6617
.7890	.5047	.7390	.5521	.6890	.6045	.6390	.6629
.7880	.5056	.7380	.5531	.6880	.6056	.6380	.6641
.7870	.5065	.7370	.5541	.6870	.6068	.6370	.6654
.7860	.5074	.7360	.5551	.6860	.6079	.6360	.6666
.7850	.5083	.7350	.5561	.6850	.6090	.6350	.6678
.7840	.5092	.7340	.5571	.6840	.6101	.6340	.6691
.7830	.5101	.7330	.5581	.6830	.6112	.6330	.6703
.7820	.5110	.7320	.5591	.6820	.6123	.6320	.6716
.7810	.5120	.7310	.5601	.6810	.6135	.6310	.6728
.7800	.5129	.7300	.5611	.6800	.6146	.6300	.6741
.7790	.5138	.7290	.5622	.6790	.6157	.6290	.6754
.7780	.5147	.7280	.5632	.6780	.6168	.6280	.6766
.7770	.5156	.7270	.5642	.6770	.6180	.6270	.6779
.7760	.5166	.7260	.5652	.6760	.6191	.6260	.6792
.7750	.5175	.7250	.5662	.6750	.6202	.6250	.6804
.7740	.5184	.7240	.5673	.6740	.6214	.6240	.6817
.7730	.5193	.7230	.5683	.6730	.6225	.6230	.6830
.7720	.5203	.7220	.5693	.6720	.6237	.6220	.6843
.7710	.5212	.7210	.5703	.6710	.6248	.6210	.6855
.7700	.5221	.7200	.5714	.6700	.6260	.6200	.6868
.7690	.5231	.7190	.5724	.6690	.6271	.6190	.6881
.7680	.5240	.7180	.5735	.6680	.6283	.6180	.6894
.7670	.5250	.7170	.5745	.6670	.6294	.6170	.6907
.7660	.5259	.7160	.5755	.6660	.6306	.6160	.6920
.7650	.5269	.7150	.5766	.6650	.6318	.6150	.6933
.7640	.5278	.7140	.5776	.6640	.6329	.6140	.6946
.7630	.5287	.7130	.5787	.6630	.6341	.6130	.6959
.7620	.5297	.7120	.5797	.6620	.6353	.6120	.6972
.7610	.5307	.7110	.5808	.6610	.6364	.6110	.6985
.7600	.5316	.7100	.5818	.6600	.6376	.6100	.6998
.7590	.5326	.7090	.5829	.6590	.6388	.6090	.7012
.7580	.5335	.7080	.5840	.6580	.6400	.6080	.7025
.7570	.5345	.7070	.5850	.6570	.6411	.6070	.7038
.7560	.5354	.7060	.5861	.6560	.6423	.6060	.7051
.7550	.5364	.7050	.5872	.6550	.6435	.6050	.7065
.7540	.5374	.7040	.5882	.6540	.6447	.6040	.7078
.7530	.5383	.7030	.5893	.6530	.6459	.6030	.7091
.7520	.5393	.7020	.5904	.6520	.6471	.6020	.7105
.7510	.5403	.7010	.5914	.6510	.6483	.6010	.7118
						.6000	.7132

L	ƚ	L	ƚ	L	ƚ	L	ƚ
.6000	.7132	.5500	.7848	.5000	.8660	.4500	.9526
.5990	.7145	.5490	.7863	.4990	.8677	.4490	.9543
.5980	.7159	.5480	.7879	.4980	.8695	.4480	.9561
.5970	.7172	.5470	.7894	.4970	.8712	.4470	.9578
.5960	.7186	.5460	.7909	.4960	.8729	.4460	.9595
.5950	.7200	.5450	.7925	.4950	.8747	.4450	.9613
.5940	.7213	.5440	.7940	.4940	.8764	.4440	.9630
.5930	.7227	.5430	.7956	.4930	.8781	.4430	.9647
.5920	.7241	.5420	.7971	.4920	.8799	.4420	.9665
.5910	.7254	.5410	.7987	.4910	.8816	.4410	.9682
.5900	.7268	.5400	.8002	.4900	.8833	.4400	.9699
.5890	.7282	.5390	.8018	.4890	.8851	.4390	.9717
.5880	.7296	.5380	.8034	.4880	.8868	.4380	.9734
.5870	.7310	.5370	.8049	.4870	.8885	.4370	.9751
.5860	.7324	.5360	.8065	.4860	.8902	.4360	.9768
.5850	.7338	.5350	.8081	.4850	.8920	.4350	.9786
.5840	.7352	.5340	.8097	.4840	.8937	.4340	.9803
.5830	.7366	.5330	.8113	.4830	.8954	.4330	.9820
.5820	.7380	.5320	.8128	.4820	.8972	.4320	.9838
.5810	.7394	.5310	.8144	.4810	.8989	.4310	.9855
.5800	.7408	.5300	.8160	.4800	.9006	.4300	.9872
.5790	.7422	.5290	.8176	.4790	.9024	.4290	.9890
.5780	.7436	.5280	.8192	.4780	.9041	.4280	.9907
.5770	.7451	.5270	.8209	.4770	.9058	.4270	.9924
.5760	.7465	.5260	.8225	.4760	.9076	.4260	.9942
.5750	.7479	.5250	.8241	.4750	.9093	.4250	.9959
.5740	.7493	.5240	.8257	.4740	.9110	.4240	.9976
.5730	.7508	.5230	.8273	.4730	.9128	.4230	.9994
.5720	.7522	.5220	.8290	.4720	.9145	.4220	1.0011
.5710	.7537	.5210	.8306	.4710	.9162	.4210	1.0028
.5700	.7551	.5200	.8323	.4700	.9180	.4200	1.0046
.5690	.7566	.5190	.8339	.4690	.9197	.4190	1.0063
.5680	.7580	.5180	.8356	.4680	.9214	.4180	1.0080
.5670	.7595	.5170	.8372	.4670	.9232	.4170	1.0098
.5660	.7609	.5160	.8389	.4660	.9249	.4160	1.0115
.5650	.7624	.5150	.8405	.4650	.9266	.4150	1.0132
.5640	.7639	.5140	.8422	.4640	.9284	.4140	1.0150
.5630	.7653	.5130	.8439	.4630	.9301	.4130	1.0167
.5620	.7668	.5120	.8455	.4620	.9318	.4120	1.0184
.5610	.7683	.5110	.8472	.4610	.9335	.4110	1.0202
.5600	.7698	.5100	.8489	.4600	.9353	.4100	1.0219
.5590	.7713	.5090	.8506	.4590	.9370	.4090	1.0236
.5580	.7728	.5080	.8523	.4580	.9387	.4080	1.0253
.5570	.7742	.5070	.8540	.4570	.9405	.4070	1.0271
.5560	.7757	.5060	.8557	.4560	.9422	.4060	1.0288
.5550	.7772	.5050	.8574	.4550	.9439	.4050	1.0305
.5540	.7788	.5040	.8591	.4540	.9457	.4040	1.0323
.5530	.7803	.5030	.8608	.4530	.9474	.4030	1.0340
.5520	.7818	.5020	.8625	.4520	.9491	.4020	1.0357
.5510	.7833	.5010	.8643	.4510	.9509	.4010	1.0375
						.4000	1.0392

L	k	L	k	L	k	L	k
.4000	1.0392	.3500	1.1258	.3000	1.2124	.2500	1.2990
.3990	1.0409	.3490	1.1275	.2990	1.2141	.2490	1.3007
.3980	1.0427	.3480	1.1293	.2980	1.2159	.2480	1.3025
.3970	1.0444	.3470	1.1310	.2970	1.2176	.2470	1.3042
.3960	1.0461	.3460	1.1327	.2960	1.2193	.2460	1.3059
.3950	1.0479	.3450	1.1345	.2950	1.2211	.2450	1.3077
.3940	1.0496	.3440	1.1362	.2940	1.2228	.2440	1.3094
.3930	1.0513	.3430	1.1379	.2930	1.2245	.2430	1.3111
.3920	1.0531	.3420	1.1397	.2920	1.2263	.2420	1.3129
.3910	1.0548	.3410	1.1414	.2910	1.2280	.2410	1.3146
.3900	1.0565	.3400	1.1431	.2900	1.2297	.2400	1.3163
.3890	1.0583	.3390	1.1449	.2890	1.2315	.2390	1.3181
.3880	1.0600	.3380	1.1466	.2880	1.2332	.2380	1.3198
.3870	1.0617	.3370	1.1483	.2870	1.2349	.2370	1.3215
.3860	1.0635	.3360	1.1501	.2860	1.2367	.2360	1.3233
.3850	1.0652	.3350	1.1518	.2850	1.2384	.2350	1.3250
.3840	1.0669	.3340	1.1535	.2840	1.2401	.2340	1.3267
.3830	1.0686	.3330	1.1553	.2830	1.2419	.2330	1.3285
.3820	1.0704	.3320	1.1570	.2820	1.2436	.2320	1.3302
.3810	1.0721	.3310	1.1587	.2810	1.2453	.2310	1.3319
.3800	1.0738	.3300	1.1604	.2800	1.2470	.2300	1.3337
.3790	1.0756	.3290	1.1622	.2790	1.2488	.2290	1.3354
.3780	1.0773	.3280	1.1639	.2780	1.2505	.2280	1.3371
.3770	1.0790	.3270	1.1656	.2770	1.2522	.2270	1.3388
.3760	1.0808	.3260	1.1674	.2760	1.2540	.2260	1.3406
.3750	1.0825	.3250	1.1691	.2750	1.2557	.2250	1.3423
.3740	1.0842	.3240	1.1708	.2740	1.2574	.2240	1.3440
.3730	1.0860	.3230	1.1726	.2730	1.2592	.2230	1.3458
.3720	1.0877	.3220	1.1743	.2720	1.2609	.2220	1.3475
.3710	1.0894	.3210	1.1760	.2710	1.2626	.2210	1.3492
.3700	1.0912	.3200	1.1778	.2700	1.2644	.2200	1.3510
.3690	1.0929	.3190	1.1795	.2690	1.2661	.2190	1.3527
.3680	1.0946	.3180	1.1812	.2680	1.2678	.2180	1.3544
.3670	1.0964	.3170	1.1830	.2670	1.2696	.2170	1.3562
.3660	1.0981	.3160	1.1847	.2660	1.2713	.2160	1.3579
.3650	1.0998	.3150	1.1864	.2650	1.2730	.2150	1.3596
.3640	1.1016	.3140	1.1882	.2640	1.2748	.2140	1.3614
.3630	1.1033	.3130	1.1899	.2630	1.2765	.2130	1.3631
.3620	1.1050	.3120	1.1916	.2620	1.2782	.2120	1.3648
.3610	1.1068	.3110	1.1934	.2610	1.2800	.2110	1.3666
.3600	1.1085	.3100	1.1951	.2600	1.2817	.2100	1.3683
.3590	1.1102	.3090	1.1968	.2590	1.2834	.2090	1.3700
.3580	1.1119	.3080	1.1986	.2580	1.2852	.2080	1.3718
.3570	1.1137	.3070	1.2003	.2570	1.2869	.2070	1.3735
.3560	1.1154	.3060	1.2020	.2560	1.2886	.2060	1.3752
.3550	1.1171	.3050	1.2037	.2550	1.2904	.2050	1.3770
.3540	1.1189	.3040	1.2055	.2540	1.2921	.2040	1.3787
.3530	1.1206	.3030	1.2072	.2530	1.2938	.2030	1.3804
.3520	1.1223	.3020	1.2089	.2520	1.2955	.2020	1.3821
.3510	1.1241	.3010	1.2107	.2510	1.2973	.2010	1.3839
						.2000	1.3856

L	k	L	k	L	k	L	k
.2000	1.3856	.1500	1.4722	.1000	1.6062	.0500	1.7678
.1990	1.3873	.1490	1.4739	.0990	1.6085	.0490	1.7723
.1980	1.3891	.1480	1.4757	.0980	1.6109	.0480	1.7770
.1970	1.3908	.1470	1.4774	.0970	1.6133	.0470	1.7817
.1960	1.3925	.1460	1.4791	.0960	1.6157	.0460	1.7865
.1950	1.3943	.1450	1.4809	.0950	1.6182	.0450	1.7914
.1940	1.3960	.1440	1.4826	.0940	1.6207	.0440	1.7964
.1930	1.3977	.1430	1.4843	.0930	1.6232	.0430	1.8015
.1920	1.3995	.1420	1.4861	.0920	1.6257	.0420	1.8066
.1910	1.4012	.1410	1.4878	.0910	1.6283	.0410	1.8119
.1900	1.4029	.1400	1.4895	.0900	1.6309	.0400	1.8173
.1890	1.4047	.1390	1.4913	.0890	1.6335	.0390	1.8228
.1880	1.4064	.1380	1.4930	.0880	1.6362	.0380	1.8284
.1870	1.4081	.1370	1.4947	.0870	1.6389	.0370	1.8341
.1860	1.4099	.1360	1.4965	.0860	1.6416	.0360	1.8399
.1850	1.4116	.1350	1.4982	.0850	1.6444	.0350	1.8459
.1840	1.4133	.1340	1.4999	.0840	1.6472	.0340	1.8520
.1830	1.4151	.1330	1.5017	.0830	1.6500	.0330	1.8582
.1820	1.4168	.1320	1.5034	.0820	1.6528	.0320	1.8646
.1810	1.4185	.1310	1.5051	.0810	1.6557	.0310	1.8711
.1800	1.4203	.1300	1.5069	.0800	1.6587	.0300	1.8777
.1790	1.4220	.1290	1.5086	.0790	1.6616	.0290	1.8846
.1780	1.4237	.1280	1.5103	.0780	1.6646	.0280	1.8916
.1770	1.4254	.1270	1.5507	.0770	1.6677	.0270	1.8988
.1760	1.4272	.1260	1.5526	.0760	1.6707	.0260	1.9062
.1750	1.4289	.1250	1.5544	.0750	1.6739	.0250	1.9137
.1740	1.4306	.1240	1.5562	.0740	1.6770	.0240	1.9215
.1730	1.4324	.1230	1.5581	.0730	1.6802	.0230	1.9296
.1720	1.4341	.1220	1.5600	.0720	1.6834	.0220	1.9378
.1710	1.4358	.1210	1.5618	.0710	1.6867	.0210	1.9464
.1700	1.4376	.1200	1.5638	.0700	1.6901	.0200	1.9552
.1690	1.4393	.1190	1.5657	.0690	1.6934	.0190	1.9643
.1680	1.4410	.1180	1.5676	.0680	1.6969	.0180	1.9737
.1670	1.4428	.1170	1.5696	.0670	1.7003	.0170	1.9834
.1660	1.4445	.1160	1.5716	.0660	1.7038	.0160	1.9936
.1650	1.4462	.1150	1.5736	.0650	1.7074	.0150	2.0041
.1640	1.4480	.1140	1.5756	.0640	1.7110	.0140	2.0151
.1630	1.4497	.1130	1.5777	.0630	1.7147	.0130	2.0266
.1620	1.4514	.1120	1.5797	.0620	1.7184	.0120	2.0386
.1610	1.4532	.1110	1.5818	.0610	1.7222	.0110	2.0512
.1600	1.4549	.1100	1.5839	.0600	1.7260	.0100	2.0646
.1590	1.4566	.1090	1.5860	.0590	1.7299	.0090	2.0787
.1580	1.4584	.1080	1.5882	.0580	1.7338	.0080	2.0938
.1570	1.4601	.1070	1.5903	.0570	1.7379	.0070	2.1100
.1560	1.4618	.1060	1.5925	.0560	1.7419	.0060	2.1276
.1550	1.4636	.1050	1.5947	.0550	1.7461	.0050	2.1468
.1540	1.4653	.1040	1.5970	.0540	1.7503	.0040	2.1683
.1530	1.4670	.1030	1.5992	.0530	1.7545	.0030	2.1929
.1520	1.4688	.1020	1.6015	.0520	1.7589	.0020	2.2223
.1510	1.4705	.1010	1.6038	.0510	1.7633	.0010	2.2608
						.0000	2.3441

TABLES OF ξ VERSUS L

ξ	L	ξ	L	ξ	L	ξ	L
.0000	2.0000	.0250	1.8772	.0500	1.7671	.0750	1.6678
.0005	1.9974	.0255	1.8749	.0505	1.7650	.0755	1.6659
.0010	1.9948	.0260	1.8726	.0510	1.7629	.0760	1.6641
.0015	1.9922	.0265	1.8702	.0515	1.7608	.0765	1.6622
.0020	1.9897	.0270	1.8679	.0520	1.7588	.0770	1.6603
.0025	1.9871	.0275	1.8656	.0525	1.7567	.0775	1.6584
.0030	1.9845	.0280	1.8633	.0530	1.7546	.0780	1.6566
.0035	1.9820	.0285	1.8611	.0535	1.7526	.0785	1.6547
.0040	1.9794	.0290	1.8588	.0540	1.7505	.0790	1.6528
.0045	1.9769	.0295	1.8565	.0545	1.7485	.0795	1.6510
.0050	1.9743	.0300	1.8542	.0550	1.7464	.0800	1.6491
.0055	1.9718	.0305	1.8519	.0555	1.7444	.0805	1.6473
.0060	1.9692	.0310	1.8497	.0560	1.7423	.0810	1.6454
.0065	1.9667	.0315	1.8474	.0565	1.7403	.0815	1.6436
.0070	1.9642	.0320	1.8452	.0570	1.7383	.0820	1.6418
.0075	1.9617	.0325	1.8429	.0575	1.7362	.0825	1.6399
.0080	1.9592	.0330	1.8407	.0580	1.7342	.0830	1.6381
.0085	1.9567	.0335	1.8384	.0585	1.7322	.0835	1.6363
.0090	1.9542	.0340	1.8362	.0590	1.7302	.0840	1.6344
.0095	1.9517	.0345	1.8340	.0595	1.7282	.0845	1.6326
.0100	1.9492	.0350	1.8317	.0600	1.7262	.0850	1.6308
.0105	1.9467	.0355	1.8295	.0605	1.7242	.0855	1.6290
.0110	1.9443	.0360	1.8273	.0610	1.7222	.0860	1.6272
.0115	1.9418	.0365	1.8251	.0615	1.7202	.0865	1.6254
.0120	1.9393	.0370	1.8229	.0620	1.7182	.0870	1.6236
.0125	1.9369	.0375	1.8207	.0625	1.7162	.0875	1.6218
.0130	1.9344	.0380	1.8185	.0630	1.7142	.0880	1.6200
.0135	1.9320	.0385	1.8163	.0635	1.7122	.0885	1.6182
.0140	1.9295	.0390	1.8141	.0640	1.7103	.0890	1.6164
.0145	1.9271	.0395	1.8119	.0645	1.7083	.0895	1.6146
.0150	1.9247	.0400	1.8097	.0650	1.7063	.0900	1.6128
.0155	1.9222	.0405	1.8076	.0655	1.7044	.0905	1.6110
.0160	1.9198	.0410	1.8054	.0660	1.7024	.0910	1.6093
.0165	1.9174	.0415	1.8032	.0665	1.7005	.0915	1.6075
.0170	1.9150	.0420	1.8011	.0670	1.6985	.0920	1.6057
.0175	1.9126	.0425	1.7989	.0675	1.6966	.0925	1.6040
.0180	1.9102	.0430	1.7967	.0680	1.6946	.0930	1.6022
.0185	1.9078	.0435	1.7946	.0685	1.6927	.0935	1.6004
.0190	1.9054	.0440	1.7925	.0690	1.6907	.0940	1.5987
.0195	1.9030	.0445	1.7903	.0695	1.6888	.0945	1.5969
.0200	1.9007	.0450	1.7882	.0700	1.6869	.0950	1.5952
.0205	1.8983	.0455	1.7861	.0705	1.6850	.0955	1.5934
.0210	1.8959	.0460	1.7839	.0710	1.6830	.0960	1.5917
.0215	1.8936	.0465	1.7818	.0715	1.6811	.0965	1.5900
.0220	1.8912	.0470	1.7797	.0720	1.6792	.0970	1.5882
.0225	1.8889	.0475	1.7776	.0725	1.6773	.0975	1.5865
.0230	1.8865	.0480	1.7755	.0730	1.6754	.0980	1.5848
.0235	1.8842	.0485	1.7734	.0735	1.6735	.0985	1.5830
.0240	1.8818	.0490	1.7713	.0740	1.6716	.0990	1.5813
.0245	1.8795	.0495	1.7692	.0745	1.6697	.0995	1.5796
						.1000	1.5779

k	L	k	L	k	L	k	L
.1000	1.5779	.1250	1.4960	.1500	1.4211	.1750	1.3524
.1005	1.5762	.1255	1.4944	.1505	1.4197	.1755	1.3511
.1010	1.5744	.1260	1.4929	.1510	1.4182	.1760	1.3498
.1015	1.5727	.1265	1.4913	.1515	1.4168	.1765	1.3485
.1020	1.5710	.1270	1.4897	.1520	1.4154	.1770	1.3471
.1025	1.5693	.1275	1.4882	.1525	1.4140	.1775	1.3458
.1030	1.5676	.1280	1.4866	.1530	1.4126	.1780	1.3445
.1035	1.5659	.1285	1.4851	.1535	1.4111	.1785	1.3432
.1040	1.5643	.1290	1.4836	.1540	1.4097	.1790	1.3419
.1045	1.5626	.1295	1.4820	.1545	1.4083	.1795	1.3406
.1050	1.5609	.1300	1.4805	.1550	1.4069	.1800	1.3393
.1055	1.5592	.1305	1.4789	.1555	1.4055	.1805	1.3380
.1060	1.5575	.1310	1.4774	.1560	1.4041	.1810	1.3367
.1065	1.5558	.1315	1.4759	.1565	1.4027	.1815	1.3354
.1070	1.5542	.1320	1.4743	.1570	1.4013	.1820	1.3342
.1075	1.5525	.1325	1.4728	.1575	1.3999	.1825	1.3329
.1080	1.5508	.1330	1.4713	.1580	1.3985	.1830	1.3316
.1085	1.5492	.1335	1.4698	.1585	1.3971	.1835	1.3303
.1090	1.5475	.1340	1.4683	.1590	1.3957	.1840	1.3290
.1095	1.5459	.1345	1.4668	.1595	1.3943	.1845	1.3277
.1100	1.5442	.1350	1.4652	.1600	1.3929	.1850	1.3265
.1105	1.5426	.1355	1.4637	.1605	1.3915	.1855	1.3252
.1110	1.5409	.1360	1.4622	.1610	1.3902	.1860	1.3239
.1115	1.5393	.1365	1.4607	.1615	1.3888	.1865	1.3227
.1120	1.5376	.1370	1.4592	.1620	1.3874	.1870	1.3214
.1125	1.5360	.1375	1.4577	.1625	1.3860	.1875	1.3201
.1130	1.5344	.1380	1.4562	.1630	1.3847	.1880	1.3189
.1135	1.5327	.1385	1.4547	.1635	1.3833	.1885	1.3176
.1140	1.5311	.1390	1.4532	.1640	1.3819	.1890	1.3163
.1145	1.5295	.1395	1.4518	.1645	1.3806	.1895	1.3151
.1150	1.5278	.1400	1.4503	.1650	1.3792	.1900	1.3138
.1155	1.5262	.1405	1.4488	.1655	1.3778	.1905	1.3126
.1160	1.5246	.1410	1.4473	.1660	1.3765	.1910	1.3113
.1165	1.5230	.1415	1.4458	.1665	1.3751	.1915	1.3101
.1170	1.5214	.1420	1.4444	.1670	1.3738	.1920	1.3088
.1175	1.5198	.1425	1.4429	.1675	1.3724	.1925	1.3076
.1180	1.5182	.1430	1.4414	.1680	1.3711	.1930	1.3063
.1185	1.5166	.1435	1.4400	.1685	1.3697	.1935	1.3051
.1190	1.5150	.1440	1.4385	.1690	1.3684	.1940	1.3038
.1195	1.5134	.1445	1.4370	.1695	1.3670	.1945	1.3026
.1200	1.5118	.1450	1.4356	.1700	1.3657	.1950	1.3014
.1205	1.5102	.1455	1.4341	.1705	1.3643	.1955	1.3001
.1210	1.5086	.1460	1.4327	.1710	1.3630	.1960	1.2989
.1215	1.5070	.1465	1.4312	.1715	1.3617	.1965	1.2977
.1220	1.5054	.1470	1.4298	.1720	1.3603	.1970	1.2964
.1225	1.5038	.1475	1.4283	.1725	1.3590	.1975	1.2952
.1230	1.5023	.1480	1.4269	.1730	1.3577	.1980	1.2940
.1235	1.5007	.1485	1.4254	.1735	1.3564	.1985	1.2928
.1240	1.4991	.1490	1.4240	.1740	1.3550	.1990	1.2915
.1245	1.4975	.1495	1.4225	.1745	1.3537	.1995	1.2903
						.2000	1.2891

ξ	L	ξ	L	ξ	L	ξ	L
.2000	1.2891	.2250	1.2306	.2500	1.1764	.2750	1.1261
.2005	1.2879	.2255	1.2295	.2505	1.1754	.2755	1.1251
.2010	1.2867	.2260	1.2284	.2510	1.1744	.2760	1.1241
.2015	1.2855	.2265	1.2273	.2515	1.1733	.2765	1.1232
.2020	1.2843	.2270	1.2262	.2520	1.1723	.2770	1.1222
.2025	1.2831	.2275	1.2250	.2525	1.1712	.2775	1.1212
.2030	1.2819	.2280	1.2239	.2530	1.1702	.2780	1.1203
.2035	1.2807	.2285	1.2228	.2535	1.1692	.2785	1.1193
.2040	1.2794	.2290	1.2217	.2540	1.1681	.2790	1.1183
.2045	1.2783	.2295	1.2206	.2545	1.1671	.2795	1.1174
.2050	1.2771	.2300	1.2195	.2550	1.1661	.2800	1.1164
.2055	1.2759	.2305	1.2184	.2555	1.1650	.2805	1.1155
.2060	1.2747	.2310	1.2173	.2560	1.1640	.2810	1.1145
.2065	1.2735	.2315	1.2162	.2565	1.1630	.2815	1.1136
.2070	1.2723	.2320	1.2151	.2570	1.1620	.2820	1.1126
.2075	1.2711	.2325	1.2140	.2575	1.1610	.2825	1.1117
.2080	1.2699	.2330	1.2129	.2580	1.1599	.2830	1.1107
.2085	1.2687	.2335	1.2118	.2585	1.1589	.2835	1.1098
.2090	1.2675	.2340	1.2107	.2590	1.1579	.2840	1.1088
.2095	1.2664	.2345	1.2096	.2595	1.1569	.2845	1.1079
.2100	1.2652	.2350	1.2085	.2600	1.1559	.2850	1.1069
.2105	1.2640	.2355	1.2074	.2605	1.1548	.2855	1.1060
.2110	1.2628	.2360	1.2063	.2610	1.1538	.2860	1.1050
.2115	1.2617	.2365	1.2052	.2615	1.1528	.2865	1.1041
.2120	1.2605	.2370	1.2041	.2620	1.1518	.2870	1.1031
.2125	1.2593	.2375	1.2030	.2625	1.1508	.2875	1.1022
.2130	1.2581	.2380	1.2020	.2630	1.1498	.2880	1.1013
.2135	1.2570	.2385	1.2009	.2635	1.1488	.2885	1.1003
.2140	1.2558	.2390	1.1998	.2640	1.1478	.2890	1.0994
.2145	1.2546	.2395	1.1987	.2645	1.1468	.2895	1.0985
.2150	1.2535	.2400	1.1976	.2650	1.1458	.2900	1.0975
.2155	1.2523	.2405	1.1966	.2655	1.1448	.2905	1.0966
.2160	1.2512	.2410	1.1955	.2660	1.1438	.2910	1.0957
.2165	1.2500	.2415	1.1944	.2665	1.1428	.2915	1.0947
.2170	1.2489	.2420	1.1933	.2670	1.1418	.2920	1.0938
.2175	1.2477	.2425	1.1923	.2675	1.1408	.2925	1.0929
.2180	1.2466	.2430	1.1912	.2680	1.1398	.2930	1.0920
.2185	1.2454	.2435	1.1901	.2685	1.1383	.2935	1.0910
.2190	1.2443	.2440	1.1891	.2690	1.1378	.2940	1.0901
.2195	1.2431	.2445	1.1880	.2695	1.1368	.2945	1.0892
.2200	1.2420	.2450	1.1870	.2700	1.1359	.2950	1.0883
.2205	1.2408	.2455	1.1859	.2705	1.1349	.2955	1.0874
.2210	1.2397	.2460	1.1848	.2710	1.1339	.2960	1.0864
.2215	1.2386	.2465	1.1838	.2715	1.1329	.2965	1.0855
.2220	1.2374	.2470	1.1827	.2720	1.1319	.2970	1.0846
.2225	1.2363	.2475	1.1817	.2725	1.1310	.2975	1.0837
.2230	1.2352	.2480	1.1806	.2730	1.1300	.2980	1.0828
.2235	1.2340	.2485	1.1796	.2735	1.1290	.2985	1.0819
.2240	1.2329	.2490	1.1785	.2740	1.1280	.2990	1.0810
.2245	1.2318	.2495	1.1775	.2745	1.1270	.2995	1.0801
						.3000	1.0791

k	L	k	L	k	L	k	L
.3000	1.0791	.3250	1.0353	.3500	.9943	.3750	.9558
.3005	1.0782	.3255	1.0345	.3505	.9935	.3755	.9550
.3010	1.0773	.3260	1.0336	.3510	.9927	.3760	.9543
.3015	1.0764	.3265	1.0328	.3515	.9919	.3765	.9535
.3020	1.0755	.3270	1.0319	.3520	.9911	.3770	.9528
.3025	1.0746	.3275	1.0311	.3525	.9903	.3775	.9521
.3030	1.0737	.3280	1.0302	.3530	.9895	.3780	.9513
.3035	1.0728	.3285	1.0294	.3535	.9887	.3785	.9506
.3040	1.0719	.3290	1.0286	.3540	.9879	.3790	.9498
.3045	1.0710	.3295	1.0277	.3545	.9872	.3795	.9491
.3050	1.0701	.3300	1.0269	.3550	.9864	.3800	.9484
.3055	1.0692	.3305	1.0260	.3555	.9856	.3805	.9476
.3060	1.0684	.3310	1.0252	.3560	.9848	.3810	.9469
.3065	1.0675	.3315	1.0244	.3565	.9840	.3815	.9462
.3070	1.0666	.3320	1.0235	.3570	.9832	.3820	.9454
.3075	1.0657	.3325	1.0227	.3575	.9825	.3825	.9447
.3080	1.0648	.3330	1.0219	.3580	.9817	.3830	.9440
.3085	1.0639	.3335	1.0211	.3585	.9809	.3835	.9432
.3090	1.0630	.3340	1.0202	.3590	.9801	.3840	.9425
.3095	1.0621	.3345	1.0194	.3595	.9794	.3845	.9418
.3100	1.0613	.3350	1.0186	.3600	.9786	.3850	.9410
.3105	1.0604	.3355	1.0177	.3605	.9778	.3855	.9403
.3110	1.0595	.3360	1.0169	.3610	.9770	.3860	.9396
.3115	1.0586	.3365	1.0161	.3615	.9763	.3865	.9389
.3120	1.0577	.3370	1.0153	.3620	.9755	.3870	.9381
.3125	1.0569	.3375	1.0145	.3625	.9747	.3875	.9374
.3130	1.0560	.3380	1.0136	.3630	.9740	.3880	.9367
.3135	1.0551	.3385	1.0128	.3635	.9732	.3885	.9360
.3140	1.0542	.3390	1.0120	.3640	.9724	.3890	.9352
.3145	1.0534	.3395	1.0112	.3645	.9716	.3895	.9345
.3150	1.0525	.3400	1.0104	.3650	.9709	.3900	.9338
.3155	1.0516	.3405	1.0096	.3655	.9701	.3905	.9331
.3160	1.0508	.3410	1.0087	.3660	.9694	.3910	.9324
.3165	1.0499	.3415	1.0079	.3665	.9686	.3915	.9316
.3170	1.0490	.3420	1.0071	.3670	.9678	.3920	.9309
.3175	1.0482	.3425	1.0063	.3675	.9671	.3925	.9302
.3180	1.0473	.3430	1.0055	.3680	.9663	.3930	.9295
.3185	1.0464	.3435	1.0047	.3685	.9656	.3935	.9288
.3190	1.0456	.3440	1.0039	.3690	.9648	.3940	.9281
.3195	1.0447	.3445	1.0031	.3695	.9640	.3945	.9274
.3200	1.0438	.3450	1.0023	.3700	.9633	.3950	.9266
.3205	1.0430	.3455	1.0015	.3705	.9625	.3955	.9259
.3210	1.0421	.3460	1.0007	.3710	.9618	.3960	.9252
.3215	1.0413	.3465	.9999	.3715	.9610	.3965	.9245
.3220	1.0404	.3470	.9991	.3720	.9603	.3970	.9238
.3225	1.0396	.3475	.9983	.3725	.9595	.3975	.9231
.3230	1.0387	.3480	.9975	.3730	.9588	.3980	.9224
.3235	1.0379	.3485	.9967	.3735	.9580	.3985	.9217
.3240	1.0370	.3490	.9959	.3740	.9573	.3990	.9210
.3245	1.0362	.3495	.9951	.3745	.9565	.3995	.9203
						.4000	.9196

ꝁ	L	ꝁ	L	ꝁ	L	ꝁ	L
.4000	.9196	.4250	.8855	.4500	.8534	.4750	.8230
.4005	.9189	.4255	.8848	.4505	.8527	.4755	.8224
.4010	.9182	.4260	.8842	.4510	.8521	.4760	.8218
.4015	.9175	.4265	.8835	.4515	.8515	.4765	.8212
.4020	.9168	.4270	.8829	.4520	.8509	.4770	.8206
.4025	.9161	.4275	.8822	.4525	.8502	.4775	.8200
.4030	.9154	.4280	.8816	.4530	.8496	.4780	.8194
.4035	.9147	.4285	.8809	.4535	.8490	.4785	.8189
.4040	.9140	.4290	.8802	.4540	.8484	.4790	.8183
.4045	.9133	.4295	.8796	.4545	.8478	.4795	.8177
.4050	.9126	.4300	.8789	.4550	.8471	.4800	.8171
.4055	.9119	.4305	.8783	.4555	.8465	.4805	.8165
.4060	.9112	.4310	.8776	.4560	.8459	.4810	.8159
.4065	.9105	.4315	.8770	.4565	.8453	.4815	.8154
.4070	.9098	.4320	.8763	.4570	.8447	.4820	.8148
.4075	.9091	.4325	.8757	.4575	.8441	.4825	.8142
.4080	.9085	.4330	.8750	.4580	.8435	.4830	.8136
.4085	.9078	.4335	.8744	.4585	.8428	.4835	.8130
.4090	.9071	.4340	.8737	.4590	.8422	.4840	.8125
.4095	.9064	.4345	.8731	.4595	.8416	.4845	.8119
.4100	.9057	.4350	.8724	.4600	.8410	.4850	.8113
.4105	.9050	.4355	.8718	.4605	.8404	.4855	.8107
.4110	.9043	.4360	.8711	.4610	.8398	.4860	.8101
.4115	.9037	.4365	.8705	.4615	.8392	.4865	.8096
.4120	.9030	.4370	.8698	.4620	.8386	.4870	.8090
.4125	.9023	.4375	.8692	.4625	.8380	.4875	.8084
.4130	.9016	.4380	.8686	.4630	.8374	.4880	.8078
.4135	.9009	.4385	.8679	.4635	.8367	.4885	.8073
.4140	.9003	.4390	.8673	.4640	.8361	.4890	.8067
.4145	.8996	.4395	.8666	.4645	.8355	.4895	.8061
.4150	.8989	.4400	.8660	.4650	.8349	.4900	.8055
.4155	.8982	.4405	.8654	.4655	.8343	.4905	.8050
.4160	.8975	.4410	.8647	.4660	.8337	.4910	.8044
.4165	.8969	.4415	.8641	.4665	.8331	.4915	.8038
.4170	.8962	.4420	.8634	.4670	.8325	.4920	.8033
.4175	.8955	.4425	.8628	.4675	.8319	.4925	.8027
.4180	.8948	.4430	.8622	.4680	.8313	.4930	.8021
.4185	.8942	.4435	.8615	.4685	.8307	.4935	.8016
.4190	.8935	.4440	.8609	.4690	.8301	.4940	.8010
.4195	.8928	.4445	.8603	.4695	.8295	.4945	.8004
.4200	.8922	.4450	.8596	.4700	.8289	.4950	.7999
.4205	.8915	.4455	.8590	.4705	.8283	.4955	.7993
.4210	.8908	.4460	.8584	.4710	.8277	.4960	.7967
.4215	.8902	.4465	.8577	.4715	.8271	.4965	.7982
.4220	.8895	.4470	.8571	.4720	.8265	.4970	.7976
.4225	.8888	.4475	.8565	.4725	.8259	.4975	.7970
.4230	.8882	.4480	.8559	.4730	.8253	.4980	.7965
.4235	.8875	.4485	.8552	.4735	.8248	.4985	.7959
.4240	.8868	.4490	.8546	.4740	.8242	.4990	.7954
.4245	.8862	.4495	.8540	.4745	.8236	.4995	.7948
						.5000	.7942

k	L	k	L	k	L	k	L
.5000	.7942	.5500	.7411	.6000	.6931	.6500	.6496
.5010	.7931	.5510	.7401	.6010	.6922	.6510	.6488
.5020	.7920	.5520	.7391	.6020	.6913	.6520	.6479
.5030	.7909	.5530	.7381	.6030	.6904	.6530	.6471
.5040	.7898	.5540	.7371	.6040	.6895	.6540	.6463
.5050	.7887	.5550	.7361	.6050	.6886	.6550	.6455
.5060	.7876	.5560	.7351	.6060	.6877	.6560	.6446
.5070	.7865	.5570	.7341	.6070	.6868	.6570	.6438
.5080	.7854	.5580	.7331	.6080	.6859	.6580	.6430
.5090	.7843	.5590	.7321	.6090	.6850	.6590	.6422
.5100	.7832	.5600	.7311	.6100	.6841	.6600	.6414
.5110	.7821	.5610	.7301	.6110	.6832	.6610	.6406
.5120	.7810	.5620	.7292	.6120	.6823	.6620	.6397
.5130	.7799	.5630	.7282	.6130	.6814	.6630	.6389
.5140	.7788	.5640	.7272	.6140	.6805	.6640	.6381
.5150	.7777	.5650	.7262	.6150	.6796	.6650	.6373
.5160	.7766	.5660	.7252	.6160	.6788	.6660	.6365
.5170	.7755	.5670	.7243	.6170	.6779	.6670	.6357
.5180	.7745	.5680	.7233	.6180	.6770	.6680	.6349
.5190	.7734	.5690	.7223	.6190	.6761	.6690	.6341
.5200	.7723	.5700	.7214	.6200	.6752	.6700	.6333
.5210	.7712	.5710	.7204	.6210	.6744	.6710	.6325
.5220	.7702	.5720	.7194	.6220	.6735	.6720	.6317
.5230	.7691	.5730	.7185	.6230	.6726	.6730	.6309
.5240	.7680	.5740	.7175	.6240	.6717	.6740	.6301
.5250	.7670	.5750	.7165	.6250	.6709	.6750	.6293
.5260	.7659	.5760	.7156	.6260	.6700	.6760	.6285
.5270	.7649	.5770	.7146	.6270	.6691	.6770	.6277
.5280	.7638	.5780	.7137	.6280	.6683	.6780	.6269
.5290	.7628	.5790	.7127	.6290	.6674	.6790	.6261
.5300	.7617	.5800	.7118	.6300	.6665	.6800	.6254
.5310	.7607	.5810	.7108	.6310	.6657	.6810	.6246
.5320	.7596	.5820	.7099	.6320	.6648	.6820	.6238
.5330	.7586	.5830	.7089	.6330	.6639	.6830	.6230
.5340	.7575	.5840	.7080	.6340	.6631	.6840	.6222
.5350	.7565	.5850	.7070	.6350	.6622	.6850	.6214
.5360	.7554	.5860	.7061	.6360	.6614	.6860	.6207
.5370	.7544	.5870	.7052	.6370	.6605	.6870	.6199
.5380	.7534	.5880	.7042	.6380	.6597	.6880	.6191
.5390	.7523	.5890	.7033	.6390	.6588	.6890	.6183
.5400	.7513	.5900	.7024	.6400	.6580	.6900	.6176
.5410	.7503	.5910	.7014	.6410	.6571	.6910	.6168
.5420	.7492	.5920	.7005	.6420	.6563	.6920	.6160
.5430	.7482	.5930	.6996	.6430	.6555	.6930	.6152
.5440	.7472	.5940	.6987	.6440	.6546	.6940	.6145
.5450	.7462	.5950	.6977	.6450	.6538	.6950	.6137
.5460	.7452	.5960	.6968	.6460	.6529	.6960	.6129
.5470	.7442	.5970	.6959	.6470	.6521	.6970	.6122
.5480	.7431	.5980	.6950	.6480	.6513	.6980	.6114
.5490	.7421	.5990	.6941	.6490	.6504	.6990	.6107
						.7000	.6099

k	L	k	L	k	L	k	L
.7000	.6099	.7500	.5736	.8000	.5402	.8500	.5094
.7010	.6091	.7510	.5729	.8010	.5395	.8510	.5088
.7020	.6084	.7520	.5722	.8020	.5389	.8520	.5082
.7030	.6076	.7530	.5715	.8030	.5382	.8530	.5076
.7040	.6069	.7540	.5708	.8040	.5376	.8540	.5070
.7050	.6061	.7550	.5701	.8050	.5370	.8550	.5064
.7060	.6054	.7560	.5694	.8060	.5363	.8560	.5058
.7070	.6046	.7570	.5687	.8070	.5357	.8570	.5052
.7080	.6039	.7580	.5680	.8080	.5351	.8580	.5047
.7090	.6031	.7590	.5673	.8090	.5344	.8590	.5041
.7100	.6024	.7600	.5667	.8100	.5338	.8600	.5035
.7110	.6016	.7610	.5660	.8110	.5332	.8610	.5029
.7120	.6009	.7620	.5653	.8120	.5325	.8620	.5023
.7130	.6001	.7630	.5646	.8130	.5319	.8630	.5018
.7140	.5994	.7640	.5639	.8140	.5313	.8640	.5012
.7150	.5987	.7650	.5632	.8150	.5307	.8650	.5006
.7160	.5979	.7660	.5626	.8160	.5300	.8660	.5000
.7170	.5972	.7670	.5619	.8170	.5294	.8670	.4994
.7180	.5965	.7680	.5612	.8180	.5288	.8680	.4989
.7190	.5957	.7690	.5605	.8190	.5282	.8690	.4983
.7200	.5950	.7700	.5599	.8200	.5276	.8700	.4977
.7210	.5943	.7710	.5592	.8210	.5269	.8710	.4971
.7220	.5935	.7720	.5585	.8220	.5263	.8720	.4966
.7230	.5928	.7730	.5579	.8230	.5257	.8730	.4960
.7240	.5921	.7740	.5572	.8240	.5251	.8740	.4954
.7250	.5913	.7750	.5565	.8250	.5245	.8750	.4948
.7260	.5906	.7760	.5558	.8260	.5238	.8760	.4942
.7270	.5899	.7770	.5552	.8270	.5232	.8770	.4937
.7280	.5892	.7780	.5545	.8280	.5226	.8780	.4931
.7290	.5884	.7790	.5539	.8290	.5220	.8790	.4925
.7300	.5877	.7800	.5532	.8300	.5214	.8800	.4919
.7310	.5870	.7810	.5525	.8310	.5208	.8810	.4914
.7320	.5863	.7820	.5519	.8320	.5202	.8820	.4908
.7330	.5856	.7830	.5512	.8330	.5196	.8830	.4902
.7340	.5849	.7840	.5506	.8340	.5190	.8840	.4896
.7350	.5841	.7850	.5499	.8350	.5184	.8850	.4890
.7360	.5834	.7860	.5492	.8360	.5177	.8860	.4885
.7370	.5827	.7870	.5486	.8370	.5171	.8870	.4879
.7380	.5820	.7880	.5479	.8380	.5165	.8880	.4873
.7390	.5813	.7890	.5473	.8390	.5159	.8890	.4867
.7400	.5806	.7900	.5466	.8400	.5153	.8900	.4862
.7410	.5799	.7910	.5460	.8410	.5147	.8910	.4856
.7420	.5792	.7920	.5453	.8420	.5141	.8920	.4850
.7430	.5785	.7930	.5447	.8430	.5135	.8930	.4844
.7440	.5778	.7940	.5440	.8440	.5129	.8940	.4839
.7450	.5771	.7950	.5434	.8450	.5123	.8950	.4833
.7460	.5764	.7960	.5427	.8460	.5117	.8960	.4827
.7470	.5757	.7970	.5421	.8470	.5111	.8970	.4821
.7480	.5750	.7980	.5414	.8480	.5106	.8980	.4815
.7490	.5743	.7990	.5408	.8490	.5100	.8990	.4810
						.9000	.4804

k	L	k	L	k	L	k	L
.9000	.4804	.9500	.4515	1.0000	.4227	1.0500	.3938
.9010	.4798	.9510	.4509	1.0010	.4221	1.0510	.3932
.9020	.4792	.9520	.4504	1.0020	.4215	1.0520	.3926
.9030	.4787	.9530	.4498	1.0030	.4209	1.0530	.3921
.9040	.4781	.9540	.4492	1.0040	.4203	1.0540	.3915
.9050	.4775	.9550	.4486	1.0050	.4198	1.0550	.3909
.9060	.4769	.9560	.4481	1.0060	.4192	1.0560	.3903
.9070	.4763	.9570	.4475	1.0070	.4186	1.0570	.3897
.9080	.4758	.9580	.4469	1.0080	.4180	1.0580	.3892
.9090	.4752	.9590	.4463	1.0090	.4175	1.0590	.3886
.9100	.4746	.9600	.4457	1.0100	.4169	1.0600	.3880
.9110	.4740	.9610	.4452	1.0110	.4163	1.0610	.3874
.9120	.4735	.9620	.4446	1.0120	.4157	1.0620	.3869
.9130	.4729	.9630	.4440	1.0130	.4151	1.0630	.3863
.9140	.4723	.9640	.4434	1.0140	.4146	1.0640	.3857
.9150	.4717	.9650	.4429	1.0150	.4140	1.0650	.3851
.9160	.4711	.9660	.4423	1.0160	.4134	1.0660	.3845
.9170	.4706	.9670	.4417	1.0170	.4128	1.0670	.3840
.9180	.4700	.9680	.4411	1.0180	.4123	1.0680	.3834
.9190	.4694	.9690	.4406	1.0190	.4117	1.0690	.3828
.9200	.4688	.9700	.4400	1.0200	.4111	1.0700	.3822
.9210	.4683	.9710	.4394	1.0210	.4105	1.0710	.3817
.9220	.4677	.9720	.4388	1.0220	.4100	1.0720	.3811
.9230	.4671	.9730	.4382	1.0230	.4094	1.0730	.3805
.9240	.4665	.9740	.4377	1.0240	.4088	1.0740	.3799
.9250	.4660	.9750	.4371	1.0250	.4082	1.0750	.3794
.9260	.4654	.9760	.4365	1.0260	.4076	1.0760	.3788
.9270	.4648	.9770	.4359	1.0270	.4071	1.0770	.3782
.9280	.4642	.9780	.4354	1.0280	.4065	1.0780	.3776
.9290	.4636	.9790	.4348	1.0290	.4059	1.0790	.3770
.9300	.4631	.9800	.4342	1.0300	.4053	1.0800	.3765
.9310	.4625	.9810	.4336	1.0310	.4048	1.0810	.3759
.9320	.4619	.9820	.4330	1.0320	.4042	1.0820	.3753
.9330	.4613	.9830	.4325	1.0330	.4036	1.0830	.3747
.9340	.4608	.9840	.4319	1.0340	.4030	1.0840	.3742
.9350	.4602	.9850	.4313	1.0350	.4024	1.0850	.3736
.9360	.4596	.9860	.4307	1.0360	.4019	1.0860	.3730
.9370	.4590	.9870	.4302	1.0370	.4013	1.0870	.3724
.9380	.4584	.9880	.4296	1.0380	.4007	1.0880	.3718
.9390	.4579	.9890	.4290	1.0390	.4001	1.0890	.3713
.9400	.4573	.9900	.4284	1.0400	.3996	1.0900	.3707
.9410	.4567	.9910	.4278	1.0410	.3990	1.0910	.3701
.9420	.4561	.9920	.4273	1.0420	.3984	1.0920	.3695
.9430	.4556	.9930	.4267	1.0430	.3978	1.0930	.3690
.9440	.4550	.9940	.4261	1.0440	.3972	1.0940	.3684
.9450	.4544	.9950	.4255	1.0450	.3967	1.0950	.3678
.9460	.4538	.9960	.4250	1.0460	.3961	1.0960	.3672
.9470	.4533	.9970	.4244	1.0470	.3955	1.0970	.3667
.9480	.4527	.9980	.4238	1.0480	.3949	1.0980	.3661
.9490	.4521	.9990	.4232	1.0490	.3944	1.0990	.3655
						1.1000	.3649

k	L	k	L	k	L	k	L
1.1000	.3649	1.2000	.3072	1.3000	.2494	1.4000	.1917
1.1020	.3638	1.2020	.3060	1.3020	.2483	1.4020	.1906
1.1040	.3626	1.2040	.3049	1.3040	.2471	1.4040	.1894
1.1060	.3615	1.2060	.3037	1.3060	.2460	1.4060	.1882
1.1080	.3603	1.2080	.3026	1.3080	.2448	1.4080	.1871
1.1100	.3591	1.2100	.3014	1.3100	.2437	1.4100	.1859
1.1120	.3580	1.2120	.3003	1.3120	.2425	1.4120	.1848
1.1140	.3568	1.2140	.2991	1.3140	.2414	1.4140	.1836
1.1160	.3557	1.2160	.2979	1.3160	.2402	1.4160	.1825
1.1180	.3545	1.2180	.2968	1.3180	.2391	1.4180	.1813
1.1200	.3534	1.2200	.2956	1.3200	.2379	1.4200	.1802
1.1220	.3522	1.2220	.2945	1.3220	.2367	1.4220	.1790
1.1240	.3511	1.2240	.2933	1.3240	.2356	1.4240	.1779
1.1260	.3499	1.2260	.2922	1.3260	.2344	1.4260	.1767
1.1280	.3488	1.2280	.2910	1.3280	.2333	1.4280	.1755
1.1300	.3476	1.2300	.2899	1.3300	.2321	1.4300	.1744
1.1320	.3464	1.2320	.2887	1.3320	.2310	1.4320	.1732
1.1340	.3453	1.2340	.2876	1.3340	.2298	1.4340	.1721
1.1360	.3441	1.2360	.2864	1.3360	.2287	1.4360	.1709
1.1380	.3430	1.2380	.2852	1.3380	.2275	1.4380	.1698
1.1400	.3418	1.2400	.2841	1.3400	.2264	1.4400	.1686
1.1420	.3407	1.2420	.2829	1.3420	.2252	1.4420	.1675
1.1440	.3395	1.2440	.2818	1.3440	.2240	1.4440	.1663
1.1460	.3384	1.2460	.2806	1.3460	.2229	1.4460	.1652
1.1480	.3372	1.2480	.2795	1.3480	.2217	1.4480	.1640
1.1500	.3361	1.2500	.2783	1.3500	.2205	1.4500	.1628
1.1520	.3349	1.2520	.2772	1.3520	.2194	1.4520	.1617
1.1540	.3337	1.2540	.2760	1.3540	.2183	1.4540	.1605
1.1560	.3326	1.2560	.2749	1.3560	.2171	1.4560	.1594
1.1580	.3314	1.2580	.2737	1.3580	.2160	1.4580	.1582
1.1600	.3303	1.2600	.2725	1.3600	.2148	1.4600	.1571
1.1620	.3291	1.2620	.2714	1.3620	.2137	1.4620	.1559
1.1640	.3280	1.2640	.2702	1.3640	.2125	1.4640	.1548
1.1660	.3268	1.2660	.2691	1.3660	.2113	1.4660	.1536
1.1680	.3257	1.2680	.2679	1.3680	.2102	1.4680	.1525
1.1700	.3245	1.2700	.2668	1.3700	.2090	1.4700	.1513
1.1720	.3233	1.2720	.2656	1.3720	.2079	1.4720	.1501
1.1740	.3222	1.2740	.2645	1.3740	.2067	1.4740	.1490
1.1760	.3210	1.2760	.2633	1.3760	.2056	1.4760	.1478
1.1780	.3199	1.2780	.2622	1.3780	.2044	1.4780	.1467
1.1800	.3187	1.2800	.2610	1.3800	.2033	1.4800	.1455
1.1820	.3176	1.2820	.2598	1.3820	.2021	1.4820	.1444
1.1840	.3164	1.2840	.2587	1.3840	.2010	1.4840	.1432
1.1860	.3153	1.2860	.2575	1.3860	.1998	1.4860	.1421
1.1880	.3141	1.2880	.2564	1.3880	.1986	1.4880	.1409
1.1900	.3130	1.2900	.2552	1.3900	.1975	1.4900	.1398
1.1920	.3118	1.2920	.2541	1.3920	.1963	1.4920	.1386
1.1940	.3106	1.2940	.2529	1.3940	.1952	1.4940	.1374
1.1960	.3095	1.2960	.2518	1.3960	.1940	1.4960	.1363
1.1980	.3083	1.2980	.2506	1.3980	.1929	1.4980	.1351
						1.5000	.1340

ξ	L	ξ	L	ξ	L	ξ	L
1.5000	.1340	1.6250	.0923	1.7500	.0541	1.8750	.0304
1.5025	.1325	1.6275	.0913	1.7525	.0535	1.8775	.0301
1.5050	.1311	1.6300	.0904	1.7550	.0529	1.8800	.0297
1.5075	.1296	1.6325	.0894	1.7575	.0523	1.8825	.0293
1.5100	.1282	1.6350	.0885	1.7600	.0518	1.8850	.0290
1.5125	.1505	1.6375	.0875	1.7625	.0512	1.8875	.0286
1.5150	.1488	1.6400	.0866	1.7650	.0506	1.8900	.0282
1.5175	.1472	1.6425	.0857	1.7675	.0501	1.8925	.0279
1.5200	.1455	1.6450	.0848	1.7700	.0495	1.8950	.0275
1.5225	.1439	1.6475	.0839	1.7725	.0490	1.8975	.0272
1.5250	.1423	1.6500	.0830	1.7750	.0484	1.9000	.0268
1.5275	.1407	1.6525	.0821	1.7775	.0479	1.9025	.0265
1.5300	.1392	1.6550	.0813	1.7800	.0474	1.9050	.0262
1.5325	.1376	1.6575	.0804	1.7825	.0469	1.9075	.0258
1.5350	.1361	1.6600	.0796	1.7850	.0463	1.9100	.0255
1.5375	.1346	1.6625	.0787	1.7875	.0458	1.9125	.0252
1.5400	.1332	1.6650	.0779	1.7900	.0453	1.9150	.0249
1.5425	.1317	1.6675	.0771	1.7925	.0448	1.9175	.0245
1.5450	.1303	1.6700	.0763	1.7950	.0443	1.9200	.0242
1.5475	.1288	1.6725	.0755	1.7975	.0438	1.9225	.0239
1.5500	.1274	1.6750	.0747	1.8000	.0433	1.9250	.0236
1.5525	.1261	1.6775	.0739	1.8025	.0428	1.9275	.0233
1.5550	.1247	1.6800	.0731	1.8050	.0423	1.9300	.0230
1.5575	.1233	1.6825	.0723	1.8075	.0419	1.9325	.0227
1.5600	.1220	1.6850	.0715	1.8100	.0414	1.9350	.0224
1.5625	.1207	1.6875	.0706	1.8125	.0409	1.9375	.0221
1.5650	.1194	1.6900	.0700	1.8150	.0404	1.9400	.0218
1.5675	.1181	1.6925	.0693	1.8175	.0400	1.9425	.0215
1.5700	.1168	1.6950	.0686	1.8200	.0395	1.9450	.0212
1.5725	.1156	1.6975	.0676	1.8225	.0391	1.9475	.0209
1.5750	.1143	1.7000	.0671	1.8250	.0386	1.9500	.0206
1.5775	.1131	1.7025	.0664	1.8275	.0382	1.9525	.0203
1.5800	.1119	1.7050	.0657	1.8300	.0377	1.9550	.0200
1.5825	.1107	1.7075	.0650	1.8325	.0373	1.9575	.0198
1.5850	.1095	1.7100	.0643	1.8350	.0369	1.9600	.0195
1.5875	.1083	1.7125	.0636	1.8375	.0364	1.9625	.0192
1.5900	.1072	1.7150	.0629	1.8400	.0360	1.9650	.0189
1.5925	.1060	1.7175	.0623	1.8425	.0356	1.9675	.0137
1.5950	.1049	1.7200	.0616	1.8450	.0352	1.9700	.0184
1.5975	.1038	1.7225	.0609	1.8475	.0347	1.9725	.0181
1.6000	.1027	1.7250	.0603	1.8500	.0343	1.9750	.0179
1.6025	.1016	1.7275	.0596	1.8525	.0339	1.9775	.0176
1.6050	.1005	1.7300	.0590	1.8550	.0335	1.9800	.0174
1.6075	.0994	1.7325	.0584	1.8575	.0331	1.9825	.0171
1.6100	.0984	1.7350	.0577	1.8600	.0327	1.9850	.0169
1.6125	.0974	1.7375	.0571	1.8625	.0323	1.9875	.0166
1.6150	.0963	1.7400	.0565	1.8650	.0319	1.9900	.0164
1.6175	.0953	1.7425	.0559	1.8675	.0316	1.9925	.0161
1.6200	.0943	1.7450	.0553	1.8700	.0312	1.9950	.0159
1.6225	.0933	1.7475	.0547	1.8725	.0308	1.9975	.0156
						2.0000	.0154

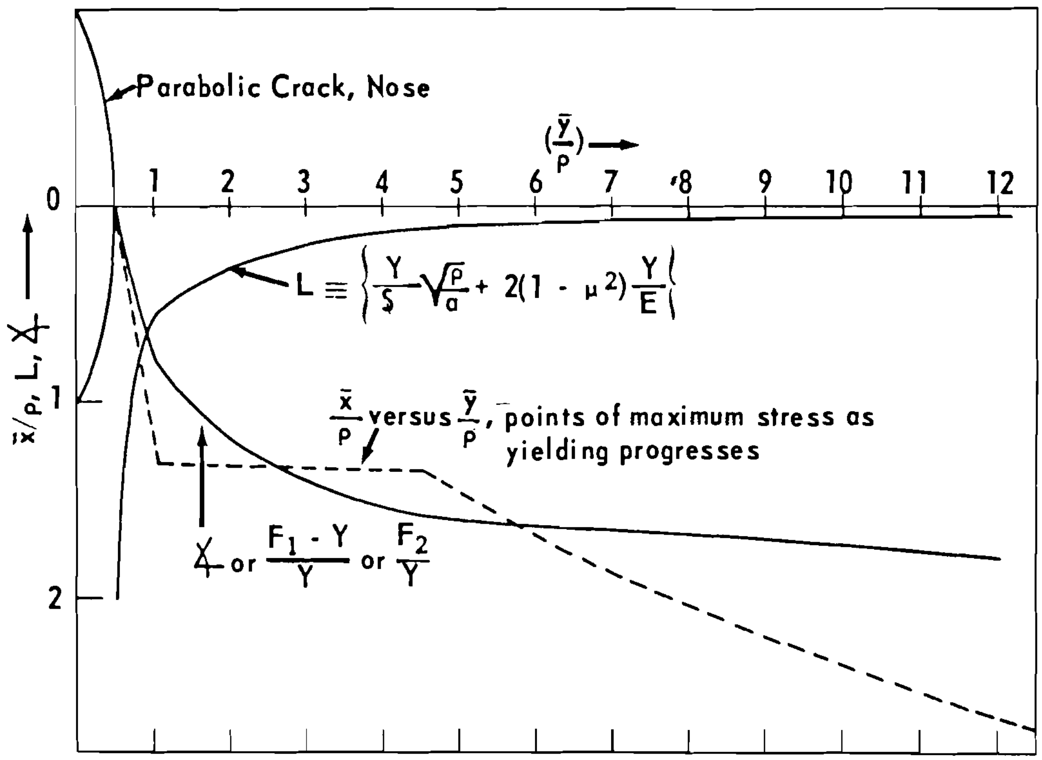
k	L
2.0000	.0154
2.0100	.0145
2.0200	.0136
2.0300	.0127
2.0400	.0119
2.0500	.0111
2.0600	.0104
2.0700	.0096
2.0800	.0089
2.0900	.0083
2.1000	.0076
2.1100	.0070
2.1200	.0064
2.1300	.0059
2.1400	.0054
2.1500	.0049
2.1600	.0044
2.1700	.0039
2.1800	.0035
2.1900	.0031
2.2000	.0028
2.2100	.0024
2.2200	.0021
2.2300	.0018
2.2400	.0015
2.2500	.0013
2.2600	.0010
2.2700	.0008
2.2800	.0006
2.2900	.0005
2.3000	.0004
2.3100	.0002
2.3200	.0001
2.3300	.0001
2.3400	.0000
2.3500	.0000

II. Formulae for L Versus ɫ, L Versus $\bar{Y}$, ɫ Versus $\bar{Y}$, and $\bar{X}$ Versus $\bar{Y}$ (Figure A-2)

$\bar{X}$ and $\bar{Y}$ are distances divided by the crack tip radius.

$|\bar{X}|$ versus $\bar{Y}$ is the locus of the maximum stress which extends beyond the crack tip and is symmetrical with respect to an axis along the center line of the crack. $\bar{X}$ is perpendicular to this line and $\bar{Y}$ is along it. $\bar{Y} = \bar{Y}_n + 0.5$ where $\bar{Y}_n$ is measured outward from the nose of the crack; thus $\bar{X}$, $\bar{Y}$ is our coordinate system. The locus is narrow, i.e., $|\bar{X}|$ versus $-\bar{Y}_n$ lies within the crack.

$$L = \frac{Y\sqrt{\pi\rho}}{S\sqrt{\pi a}} + 2(1-\mu^2)\,\frac{Y}{E}$$



PARABOLIC CRACK

L, ɫ, and Max. Stress Locus

Figure A-2.

where

$S\sqrt{\pi a}$ is the stress intensity factor K
ρ is the crack tip radius
μ is Poisson's ratio
Y is the yield strength

also $\nmid$ is the maximum trajectory angle change occurring among the shear stress trajectories traversing the yielded region corresponding to nominal loading stress S.

If the maximum stress is sufficient to cause fracture, and the weakness of the material coincides with a principal stress direction:

$$\nmid = (F_1 - Y)/Y \text{ or is}$$
$$= F_2/Y \text{ if } F_2 < F_1 - Y, \text{ where F is the fracture stress.}$$

Subscript 1 corresponds to the largest principal stress and its direction which is almost perpendicular to the crack so that the corresponding fracture surface extends the crack. Subscript 2 corresponds to the fracture perpendicular to this direction and thus to blockage of the crack, i.e., to delamination.

Three sets of formulae are required to cover the entire range of angles, $\nmid$. In each separate region the figures in vertical alignment correspond to each other:

Region 1:
$$\begin{array}{rcccl} 2 & \geq & L & \geq & 0.5 \\ 0 & \leq & \nmid & \leq & \tfrac{1}{2}\sqrt{3} \\ 0.5 & \leq & \overline{Y} & \leq & 1.1 \\ 0 & \leq & \overline{X} & \leq & 1.32 \end{array}$$

Region 2:
$$\begin{array}{rcccl} 0.5 & \geq & L & \geq & 0.1277 \\ \tfrac{1}{2}\sqrt{3} & \leq & \nmid & \leq & 1.5108 \\ 1.1 & \leq & \overline{Y} & \leq & 4.0213 \\ 1.32 & \leq & \overline{X} & \leq & 1.3505 \end{array}$$

Region 3:
$$\begin{array}{rcccl} 0.1277 & \geq & L & \geq & 0 \\ 1.5108 & \leq & \nmid & \leq & 3\pi/4 \\ 4.0213 & \leq & \overline{Y} & \leq & \infty \\ 1.3505 & \leq & \overline{X} & \leq & \infty \end{array}$$

The first derivatives of the L versus $\nmid$ relationships are equal at the junctions of the regions.

The formulae are:

Region 1: $2 \geq L \geq 0.5$
$$9/(2 - L) = \sqrt{3}/\nmid + 4$$
$$9/(2 - L) = 1/(\overline{Y} - 0.5) + 13/3$$
$$\sqrt{3}/\nmid = 1/(\overline{Y} - 0.5) + 1/3$$
$$\overline{X}/2.2 = \overline{Y} - 0.5$$

Region 2: $0.5 \geq L \geq 0.1277$

$$L = 1 - \kappa/\sqrt{3}$$
$$\sqrt{3}/\kappa = 1.2925/\overline{Y} + 0.825$$
$$1/(1 - L) = 1.2925/\overline{Y} + 0.825$$
$$97\overline{X} = \overline{Y} + 127$$

Region 3: $0.1277 \geq L \geq 0$

$$1/L = -6 + 9/(3\pi/4 - \kappa)^2$$
$$1/(L\sqrt{3}) = \overline{Y} + 1/2$$
$$3\sqrt{3}/(3\pi/4 - \kappa)^2 = \overline{Y} + (1/2 + 2\sqrt{3})$$
$$\overline{X} \cong 0.8\ (\overline{Y} - 1.1714)^{\frac{1}{2}}$$

III. GENERAL NOTATION AND RELATIONSHIPS

The relationships refer to plane strain and notches such that the nominal loading stress S is perpendicular to the notch axis.

For notches with $a/\rho \gg 1$, where "a" is the measure of depth or length and ρ the tip radius we have

$$L = 2Y/S_{tip}$$

where

S_{tip} = maximum theoretical elastic notch tip stress corrected for boundary change in shape under load, $a/\rho \gg 1$.
Y_t = simple tensile yield stress
$Y = Y_t$ for principal stress difference criterion of yielding
$Y = 2Y_t/\sqrt{3}$ for Mises-Hencky criterion of yielding

also,

$$\sqrt{\rho/a} = \sqrt{\rho_0/a} + 2(1-\mu)^2 S/E$$

where the S/E term accounts for change in shape under load (a naturally negligible term in most cases, but a machined-in radius should be such that $\sqrt{\rho_0/a} \gg 2(1-\mu)^2 S/E$ to avoid this complication, even though $\rho_0 \ll t$, the thickness, to insure essential plane strain).

S_{tip} for $a/\rho \gg 1$ is expected to be of the form (S.C.F. stands for Stress Concentration Factor)

$$S_{tip} = S\ (\text{S.C.F.})$$

with

$$\text{S.C.F.} = k\sqrt{a/\rho}$$

for notches without abrupt changes in curvature. For a wide plate under tensile loading $k = 2$ for the elliptical hole of length 2a across the loading direction; $k = 2\ (1.12147)$ for an edge crack in a wide plate under tension; $k = 4/\pi$ for a plate under tension, with a pair of deep hyperbola-shaped notches with a common axis perpendicular to the tension, the noses being a distance 2a apart; $k \cong 1$ for a round bar under tension, notched by a hyperboloid of revolution.

Hence

$L = 2Y/Sk\sqrt{a/\rho)}$, $a/\rho >> 1$

and thus $L = 2$ is the condition for incipient yielding, since incipient yielding occurs when

$Sk\sqrt{a/\rho} = Y$ (= $2Y_t/\sqrt{3}$, generally).*

Since $k = 2$ for the elliptical hole case (the case which is taken as standard in toughness testing when the ellipse is crack-like), it is seen that the 2 in the expression for L is the factor that normalizes L to the simple form

$$L = \frac{Y\sqrt{\rho}}{S\sqrt{a}}$$

for the ellipse.

When brittle fracture occurs in a direction to increase crack length, the fracture stress is associated with a stress trajectory angle change, as follows, in the L versus $\nmid$ relationship:

$\nmid = F/Y - 1$
F = nil-ductility fracture strength
Y = plane strain yield strength, $2Y_t/\sqrt{3}$.

Thus the relationship L versus $\nmid$ is

$2Y/(Sk\sqrt{a/\rho)}$ versus $(F/Y - 1)$,

a relationship between S/Y and F/Y, i.e., nominal loading stress and fracture stress, for Y and a/ρ of the specimen having the notch configuration under consideration.

The relationship between the distance $\overline{Y}/\rho$ to the point of incipient fracture is given by the tabulated formula in this Appendix, Section I.

Thus one may hope to determine fracture stress by the nominal loading stress S at incipient fracture or by the distance from the notch to the point where incipient fracture occurs, or to confirm the stress obtained by one method by that obtained by the other.

*For stress concentrators in general, we write

$M = Y/\text{Max. Elastic Notch Stress}$
$\quad = Y/[S(S.C.F.)]$

where, of course, we commonly take

$Y = 2Y_t/\sqrt{3}$

The Stress Concentration Factor is S.C.F. = 3 for a wide tension member with a small central hole. M = 1 is the condition for incipient yielding.

APPENDIX B. PROOFS, CONSTRUCTION, AND USE OF OVERLAYS

The transparent overlays are made to be used repeatedly until worn out, so long as it is desired to determine fracture stress and effective crack tip radius by use of them from yield strength and corresponding K_{Ic} measurements.

The overlays contain a curve, or curves, each identified by a parameter which conveys their relationship to a law relating fracture stress to yield strength. The curves are log-log plots of a purely mechanical stress analysis relationship existing between K_{Ic}, yield strength, modulus of elasticity, Poisson's ratio, fracture stress, and effective crack tip radius, though expressed simply in terms of dimensionless groupings of these variables called L and $\nmid$ and expressed as L versus $\nmid$. This relationship is insufficient in itself to determine fracture stress and radius in the absence of a fracture stress law relating fracture strength to yield strength and also a relation, here assumed to be determined experimentally, between K_{Ic} and yield strength.

In use the overlay is laid upon a log-log plot* of experimental or slightly modified experimental K_{Ic} versus Y data, in such a way as to best match the data in all or part of its range. In this position, the values of K and Y under the unit coordinate lines of the overlay yield the desired values of fracture stress and radius as described below. The match must include at least three experimental points, for at least three points are necessary to establish curvature which, of course, describes the shape of a curve with a continuous slope.

The overlay curves themselves are constructed from a table of L versus $\nmid$ for assumed constant values of the fracture law parameter which is here designated β and covers a practical instructive range of $-2 \le \beta \le 1$ in intervals of, say, 0.2 or 0.4 for the fracture law used here. The curves are graphs of $\log 1/(\nmid - \beta)$ versus $\log 1/[(\nmid - \beta) L]$ and are easily transferred to transparencies by a number of copy machines.† We interpret the curves in terms of fracture stress and radius.

Here, as always, the fracture stress is

$$F = Y + Y\nmid$$

so that

$$\nmid = \frac{F - Y}{Y}.$$

The fracture stress law is assumed to be linear in the K_{Ic}, Y region being matched, i.e.,

$$F = (1 + \beta)Y + f$$

so that

$$\beta = \frac{F - Y}{Y} - \frac{f}{Y} = \nmid - \frac{f}{Y}$$

and hence $\nmid - \beta = f/Y$.

*To the same scale used in constructing the overlay, attached to inside of back cover.

†For example, a 3-M Thermofax Copier.

We now distinguish two interpretations of $\log 1/[(\text{ǂ} - \beta)\,L]$ versus $\log 1/(\text{ǂ} - \beta)$ as applied, though the second contains the first which is separated out because it usually applies and is simple.

Case I: Usual Case, $L \cong Y\sqrt{\pi\rho}/K$

Our abscissa on the transparency is

$$\log 1/(\text{ǂ} - \beta) = \log Y/f = \log Y - \log f$$

so that $Y = f$ where $Y/f = 1$ and $\log Y/f = \log 1/(\text{ǂ} - \beta) = 0$.

The ordinate is

$$\log 1/[(\text{ǂ} - \beta)\,L] = \log\,[(Y/f)(K/Y\sqrt{\pi\rho})]$$
$$= \log K - \log f\sqrt{\pi\rho}$$

and $$K = f\sqrt{\pi\rho} \text{ where } 1/[(\text{ǂ} - \beta)\,L] = 1.$$

Thus if we find we can match one of the β curves of the transparent overlay to a log-log plot of experimental values of K_{Ic} versus Y, the value of the Y intersected by the unit coordinate line of the overlay will be $Y = f$. For what is log Y on the plot is [log Y - log f], i.e., 0 on the overlay when it is in the matching position. Correspondingly and similarly, the horizontal unit coordinate line of the overlay is where $K = f\sqrt{\pi\rho}$ and from this $\sqrt{\pi\rho}$ is gotten by dividing out the value of f.

We thus know $\sqrt{\pi\rho}$ as well as the constants β and f of our fracture law.

Case II: Exact Case by Iteration, $L = Y\sqrt{\pi\rho}/K + 2(1-\mu^2)Y/E$

The interpretation of the abscissa is the same as in Case I, namely $\log 1/(\text{ǂ} - \beta) = \log Y - \log f$ inasmuch as L does not enter into it.

The interpretation of the ordinate is

$$\log \frac{1}{(\text{ǂ} - \beta)L} = \log \frac{Y}{f}\,\frac{1}{Y\sqrt{\pi\rho}/K + 2(1-\mu^2)Y/E}$$
$$= \log \frac{Y}{f}\,\frac{K}{Y\sqrt{\pi\rho}}\,\frac{1}{1 + 2(1-\mu^2)K/E\sqrt{\pi\rho}}$$
$$= \log K^* - \log f\sqrt{\pi\rho}$$

where $K^* = K/[1 + 2(1-\mu^2)K/E\sqrt{\pi\rho}]$.

This indicates that we should plot log K* versus log Y, instead of log K versus log Y in making our log-log plot of experimental data as well as that $f\sqrt{\pi\rho} = K^*$ where $\log 1/[(\text{ǂ} - \beta)\,L] = \log K^*/f\sqrt{\pi\rho} = \log 1$, as explained previously in Case I.

However, $\sqrt{\pi\rho}$ in K* is an unknown and so we resort to an iteration procedure on the basis that the term in the denominator of K* involving E and $\sqrt{\pi\rho}$ can be expected to be substantially less than one.

In this procedure, we first assume this term is zero (as in Case I) and proceed as in Case I to find f and $\sqrt{\pi\rho}$. Using this $\sqrt{\pi\rho}$ as a first approximation we compute K^* from the experimental K_{Ic} and replace K by K^* on our original log-log plot of K versus Y. We then match our overlay to this data and from the values of K^* and Y under the unit lines of the overlay obtain new values of $f\sqrt{\pi\rho}$ and f. These will be an improvement over the original values though not greatly differing from them.

Further iterations may be used to improve the accuracy still further. With the last obtained value of $\sqrt{\pi\rho}$ and the *original data* values of K, new K^* values are obtained to replace any previously obtained values of K^* on the log K^* versus log Y plot and this new plot is then used to obtain new and more accurate values of f and $f\sqrt{\pi\rho}$ than those obtained previously.

However, experience shows that the scatter in original data and the rapid convergence of the procedure do not warrant more than one or two iterations.

The overlays and graphs for the examples in the text were for $\beta = 0$, i.e., $F = Y + f$, and $\beta = -1$, i.e., $F = f$, independent of Y. The curves for making these overlays and the experimental data were plotted on the largest size single-cycle log-log paper locally available that would fit into this report, with K on the vertical and Y on the horizontal scale. This size paper and the corresponding overlays are considered so generally useful that such overlays are included in this report and are called the f (for $\beta = 0$) and F (for $\beta = -1$) overlays.

However, to make an overlay to cover the larger range of experimental variables and ranges of β that may be encountered, 2×3 log-log paper, whose size also conforms to that of the report, had to be used. Such an overlay is enclosed in this report. Unlike the scale of the single-cycle paper above, this selection of paper means that for this wider range the scale is such that the Y values (plotted on the horizontal axis, as before) increase from right to left.

OTHER POSSIBLE OVERLAYS

An overlay worth noting is that gotten by plotting log L versus log $1/(\nless - \beta)$. Take $L \cong Y\sqrt{\pi\rho}/K$. Then where $L = 1$, $\sqrt{\pi\rho} = K/Y$. Also $\log L = \log Y/K + \log \sqrt{\pi\rho}$ and as treated previously $\log 1/(\nless - \beta) = \log Y - \log f$.

Thus we here plot our experimental data on the form of log Y/K versus log Y and match it with a β curve on the overlay. The line $(\nless - \beta) = 1$ on the latter then lies over the Y value for which $f = Y$ and the line $L = 1$ then lies over a Y/K value for which $\sqrt{\pi\rho} = K/Y$.

By plotting log 1/L versus log $1/(\nless - \beta)$ for the overlay, the ordinate for experimental data would be log K/Y and the line $L = 1$ would be over a K/Y value for which $\sqrt{\pi\rho} = K/Y$. As before, the line $(\nless - \beta) = 1$ would lie over f. Thus both f and $\sqrt{\pi\rho}$ are found separately rather than in combinations such as $f\sqrt{\pi\rho}$.

We see that overlays are possible other than those presented with the report. Those presented have been chosen with the idea that it is simplest to plot K versus Y on log-log paper.

DISTRIBUTION LIST

No. of Copies	To
1	Office of the Director, Defense Research and Engineering, The Pentagon, Washington, D.C. 20301
12	Commander, Defense Documentation Center, Cameron Station, Building 5, 5010 Duke Street, Alexandria, Virginia 22314
1	Metals and Ceramics Information Center, Battelle Columbus Laboratories, 505 King Avenue, Columbus, Ohio 43201
	Deputy Chief of Staff, Research, Development, and Acquisition, Headquarters Department of the Army, Washington, D. C. 20310
1	ATTN: DAMA-ARZ
	Commander, Army Research Office, P. O. Box 12211, Research Triangle Park, North Carolina 27709
1	ATTN: Information Processing Office
1	Dr. F. W. Schmiedeshoff
	Commander, U. S. Army Materiel Development and Readiness Command, 5001 Eisenhower Avenue, Alexandria, Virginia 22333
1	ATTN: DRCLDC, Mr. R. Zentner
	Commander, U. S. Army Communications Research and Development Command, Fort Monmouth, New Jersey 07703
1	ATTN: DRCDO-GG-TD
1	DRCDO-GG-DM
	Commander, U. S. Army Missile Research and Development Command, Redstone Arsenal, Alabama 35809
1	ATTN: DRDMI-RKK, Mr. C. Martens, Bldg. 7120
	Commander, U. S. Army Natick Research and Development Command, Natick, Massachusetts 01760
1	ATTN: Technical Library
1	Dr. E. W. Ross
1	DRDNA-UE, Dr. L. A. McClaine
	Commander, U. S. Army Satellite Communications Agency, Fort Monmouth, New Jersey 07703
1	ATTN: Technical Document Center
	Commander, U. S. Army Tank-Automotive Research and Development Command, Warren, Michigan 48090
1	ATTN: DRDTA-RKA
1	DRDTA-UL, Technical Library
	Commander, U. S. Army Armament Research and Development Command, Dover, New Jersey 07801
2	ATTN: Technical Library
1	DRDAR-SCM, J. D. Corrie
1	Dr. J. Fraiser
	Commander, White Sands Missile Range, New Mexico 88002
1	ATTN: STEWS-WS-VT
	Commander, Aberdeen Proving Ground, Maryland 21005
1	ATTN: STEAP-TL, Bldg. 305
	Commander, U. S. Army Armament Research and Development Command, Aberdeen Proving Ground, Maryland 21010
1	ATTN: DRDAR-QAC-E
	Commander, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland 21005
1	ATTN: Dr. R. Vitali
1	Dr. G. L. Filbey
1	Dr. W. Gillich
1	DRDAR-TSB-S (STINFO)
	Commander, Harry Diamond Laboratories, 2800 Powder Mill Road, Adelphi, Maryland 20783
1	ATTN: Technical Information Office
	Commander, Picatinny Arsenal, Dover, New Jersey 07801
1	ATTN: Mr. J. Pearson
1	G. Randers-Pehrson
1	Mr. A. Garcia
1	SARPA-RT-S
	Commander, Redstone Scientific Information Center, U. S. Army Missile Research and Development Command, Redstone Arsenal, Alabama 35809
1	ATTN: DRDMI-TB
	Commander, Watervliet Arsenal, Watervliet, New York 12189
1	ATTN: Dr. T. Davidson
1	Mr. D. P. Kendall
1	Mr. J. F. Throop
	Commander, U. S. Army Foreign Science and Technology Center, 220 7th Street, N. E., Charlottesville, Virginia 22901
1	ATTN: Mr. Marley, Military Tech
	Chief, Benet Weapons Laboratory, LCWSL, USA ARRADCOM, Watervliet Arsenal, Watervliet, New York 12189
1	ATTN: DRDAR-LCB-TL
	Director, Eustis Directorate, U. S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia 23604
1	ATTN: Mr. J. Robinson, DAVDL-E-MOS (AVRADCOM)
	U. S. Army Aviation Training Library, Fort Rucker, Alabama 36360
1	ATTN: Building 5906-5907
	Commander, U. S. Army Agency for Aviation Safety, Fort Rucker, Alabama 36362
1	ATTN: Librarian, Bldg. 4905
	Commander, USACDC Air Defense Agency, Fort Bliss, Texas 79916
1	ATTN: Technical Library
	Commander, U. S. Army Engineer School, Fort Belvoir, Virginia 22060
1	ATTN: Library
	Commander, U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi 39180
1	ATTN: Research Center Library
	Commander, U. S. Army Mobility Equipment Research and Development Center, Fort Belvoir, Virginia 22060
1	ATTN: DRDME-MW, Dr. J. W. Bond
	Commander, Naval Air Engineering Center, Lakehurst, New Jersey 08733
1	ATTN: Technical Library, Code 1115
	Director, Structural Mechanics Research, Office of Naval Research, 800 North Quincy Street, Arlington, Virginia 22203
1	ATTN: Dr. N. Perrone
	Naval Air Development Center, Aero Materials Department, Warminster, Pennsylvania 18974
1	ATTN: J. Viglione
	David Taylor Naval Ship Research and Development Laboratory, Annapolis, Maryland 21402
1	ATTN: Dr. H. P. Chu
	Naval Research Laboratory, Washington, D.C. 20375
1	ATTN: C. D. Beachem, Head, Adv. Mat'ls Tech Br. (Code 6310)
1	Dr. J. M. Krafft - Code 8430
1	E. A. Lange
1	Dr. P. P. Puzak
1	R. J. Sanford - Code 8436
1	A. M. Sullivan
1	R. W. Rice
1	S. W. Freiman
1	Dr. Jim C. I. Chang
	Chief of Naval Research, Arlington, Virginia 22217
1	ATTN: Code 471
	Naval Weapons Laboratory, Washington, D.C. 20390
1	ATTN: H. W. Romine, Mail Stop 103
1	Ship Research Committee, Maritime Transportation Research Board, National Research Council, 2101 Constitution Avenue, N. W., Washington, D.C. 20418
	Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433
2	ATTN: AFML (MXE), E. Morrissey
1	AFML (LC)
1	AFML (LLP), D. M. Forney, Jr.
1	AFML (LNC), T. J. Reinhart
1	AFML (MBC), Mr. Stanley Schulman
1	AFFDL (FB), Dr. J. C. Halpin
	Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio 45433
1	ATTN: AFFDL (FBS), C. Wallace
1	AFFDL (FBE), G. D. Sendeckyj

No. of Copies	To
	National Aeronautics and Space Administration, Washington, D.C. 20546
1	ATTN: Mr. B. G. Achhammer
1	Mr. G. C. Deutsch - Code RW
	National Aeronautics and Space Administration, Marshall Space Flight Center, Huntsville, Alabama 35812
1	ATTN: R. J. Schwinghamer, EH01, Dir., M&P Lab
1	Mr. W. A. Wilson, EH41, Bldg. 4612
	National Aeronautics and Space Administration, Langley Research Center, Hampton, Virginia 23665
1	ATTN: Mr. H. F. Hardrath, Mail Stop 188M
1	Mr. R. Foye, Mail Stop 188A
	National Aeronautics and Space Administration, Lewis Research Center, 21000 Brookpark Road, Cleveland, Ohio 44135
1	ATTN: Mr. S. S. Manson
1	Dr. J. E. Srawley, Mail Stop 105-1
1	Mr. W. F. Brown, Jr.
1	Mr. M. H. Hirschberg, Head, Fatigue Research Section, Mail Stop 49-1
	National Bureau of Standards, U. S. Department of Commerce, Washington, D.C. 20234
1	ATTN: Mr. J. A. Bennett
1	Mechanical Properties Data Center, Belfour Stulen Inc., 13917 W. Bay Shore Drive, Traverse City, Michigan 49684
1	Mr. W. F. Anderson, Atomics International, Canoga Park, California 91303
	Midwest Research Institute, 425 Coker Boulevard, Kansas City, Missouri 64110
1	ATTN: Mr. G. Gross
1	Mr. A. Hurlich, Convair Div., General Dynamics Corp., Mail Zone 630-01, P. O. Box 80847, San Diego, California 92138
1	Mr. J. G. Kaufman, Alcoa Research Laboratories, New Kensington, Pennsylvania 15068
1	Mr. P. N. Randall, TRW Systems Group - 0-1/2210, One Space Park, Redondo Beach, California 90278
1	Dr. E. A. Steigerwald, TRW Metals Division, P. O. Box 250, Minerva, Ohio 44657
1	Dr. George R. Irwin, Department of Mechanical Engineering, University of Maryland, College Park, Maryland 20742
1	Mr. W. A. Van der Sluys, Research Center, Babcock and Wilcox, Alliance, Ohio 44601
1	Mr. B. M. Wundt, 2346 Shirl Lane, Schenectady, New York 12309
	Battelle Columbus Laboratories, 505 King Avenue, Columbus, Ohio 43201
1	ATTN: Mr. J. Campbell
1	Dr. G. T. Hahn
1	R. G. Hoagland, Metal Science Group
1	Dr. E. Rybicki
	General Electric Company, Schenectady, New York 12010
1	ATTN: Mr. A. J. Brothers, Materials & Processes Laboratory
	General Electric Company, Schenectady, New York 12309
1	ATTN: Mr. S. Yukawa, Metallurgy Unit
1	Mr. E. E. Zwicky, Jr.
	General Electric Company, Knolls Atomic Power Laboratory, P. O. Box 1072, Schenectady, New York 12301
1	ATTN: Mr. F. J. Mehringer
1	Dr. L. F. Coffin, Room 1C41-K1, Corp. R&D, General Electric Company, P. O. Box 8, Schenectady, New York 12301
	United States Steel Corporation, Monroeville, Pennsylvania 15146
1	ATTN: Mr. S. R. Novak
1	Dr. A. K. Shoemaker, Research Laboratory, Mail Stop 78
	Westinghouse Electric Company, Bettis Atomic Power Laboratory, P. O. Box 109, West Mifflin, Pennsylvania 15122
1	ATTN: Mr. M. L. Parrish
	Westinghouse Research and Development Center, 1310 Beulah Road, Pittsburgh, Pennsylvania 15235
1	ATTN: Mr. E. T. Wessel
1	Mr. M. J. Manjoine
1	Dr. Alan S. Tetelman, Failure Analysis Associates, Suite 4, 11777 Mississippi Ave., Los Angeles, California 90025
	Brown University, Providence, Rhode Island 02912
1	ATTN: Prof. J. R. Rice
1	Prof. W. N. Findley, Division of Engineering, Box D
1	Prof. P. C. Paris
	Carnegie-Mellon University, Department of Mechanical Engineering, Schenley Park, Pittsburgh, Pennsylvania 15213
1	ATTN: Dr. J. L. Swedlow
1	Prof. J. D. Lubahn, Colorado School of Mines, Golden, Colorado 80401
1	Prof. J. Dvorak, Civil Engineering Department, Duke University, Durham, North Carolina 27706
	George Washington University, School of Engineering and Applied Sciences, Washington, D.C. 20052
1	ATTN: Dr. H. Liebowitz
	Lehigh University, Bethlehem, Pennsylvania 18015
1	ATTN: Prof. G. C. Sih
1	Prof. R. Roberts
1	Prof. R. P. Wei
1	Prof. F. Erodgan
	Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
1	ATTN: Prof. B. L. Averbach, Materials Center, 13-5082
1	Prof. F. A. McClintock, Room 1-304
1	Prof. R. M. Pelloux
1	Prof. T. H. H. Pian, Department of Aeronautics and Astronautics
1	Prof. A. S. Argon, Room 1-306
	Syracuse University, Department of Chemical Engineering and Metallurgy, 409 Link Hall, Syracuse, New York 13210
1	ATTN: Mr. H. W. Liu
1	Dr. V. Weiss, Metallurgical Research Labs., Bldg. D-6
1	Prof. E. R. Parker, Department of Materials Science and Engineering, University of California, Berkeley, California 94700
1	Prof. W. Goldsmith, Department of Mechanical Engineering, University of California, Berkeley, California 94720
	University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544
1	ATTN: Dr. R. Karpp
1	Prof. A. J. McEvily, Metallurgy Department U-136, University of Connecticut, Storrs, Connecticut 06268
1	Prof. D. Drucker, Dean of School of Engineering, University of Illinois, Champaign, Illinois 61820
	University of Illinois, Urbana, Illinois 61801
1	ATTN: Prof. H. T. Corten, Department of Theoretical and Applied Mechanics, 212 Talbot Laboratory
1	Prof. T. J. Dolan, Department of Theoretical and Applied Mechanics
1	Prof. J. Morrow, 321 Talbot Laboratory
1	Mr. G. M. Sinclair, Department of Theoretical and Applied Mechanics
1	Prof. R. I. Stephens, Materials Engineering Division, University of Iowa, Iowa City, Iowa 52242
1	Prof. D. K. Felbeck, Department of Mechanical Engineering, University of Michigan, 2046 East Engineering, Ann Arbor, Michigan 48109
1	Dr. M. L. Williams, Dean of Engineering, 240 Benedum Hall, University of Pittsburgh, Pittsburgh, Pennsylvania 15260
1	Prof. A. Kobayashi, Department of Mechanical Engineering, FU-10, Universityof Washington, Seattle, Washington 98195
	State University of New York at Stony Brook, Stony Brook, New York 11790
1	ATTN: Prof. Fu-Pen Chiang, Department of Mechanics
	Director, Army Materials and Mechanics Research Center, Watertown, Massachusetts 02172
2	ATTN: DRXMR-PL
1	DRXMR-AG-MD
3	Author

Constant f Overlay

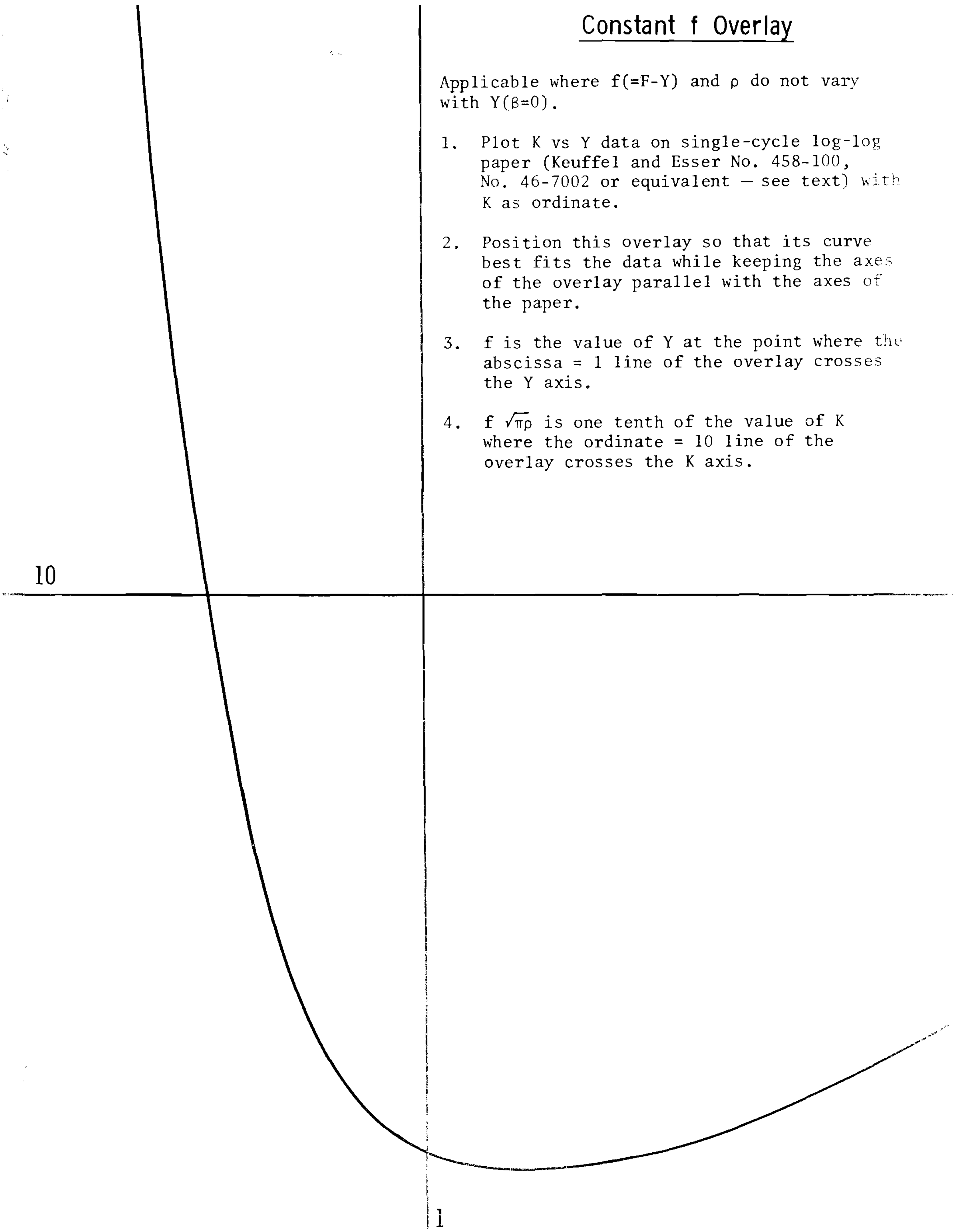
Applicable where f(=F-Y) and ρ do not vary with Y(β=0).

1. Plot K vs Y data on single-cycle log-log paper (Keuffel and Esser No. 458-100, No. 46-7002 or equivalent — see text) with K as ordinate.

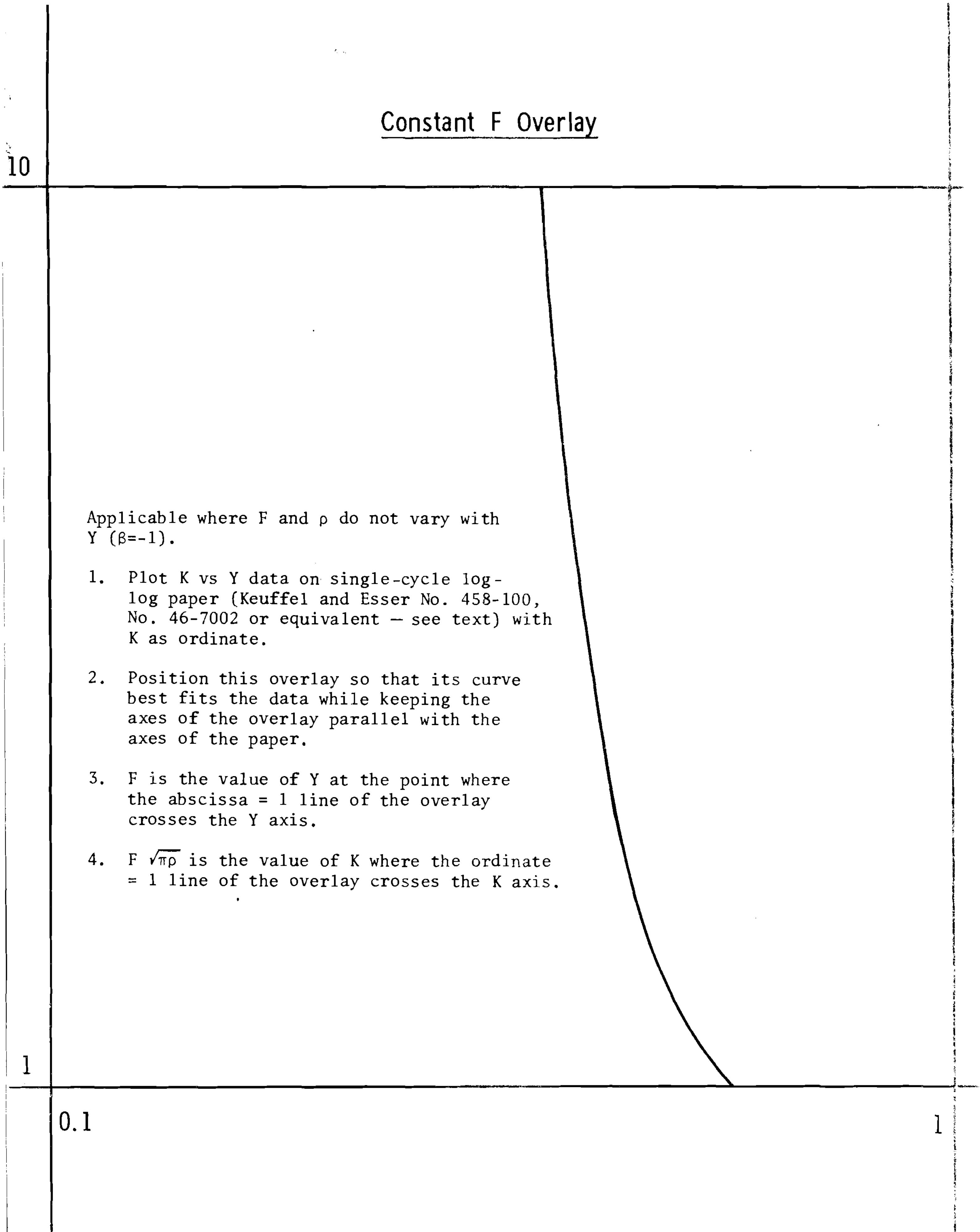
2. Position this overlay so that its curve best fits the data while keeping the axes of the overlay parallel with the axes of the paper.

3. f is the value of Y at the point where the abscissa = 1 line of the overlay crosses the Y axis.

4. f $\sqrt{\pi\rho}$ is one tenth of the value of K where the ordinate = 10 line of the overlay crosses the K axis.



Constant F Overlay



Applicable where F and ρ do not vary with Y (β=-1).

1. Plot K vs Y data on single-cycle log-log paper (Keuffel and Esser No. 458-100, No. 46-7002 or equivalent – see text) with K as ordinate.

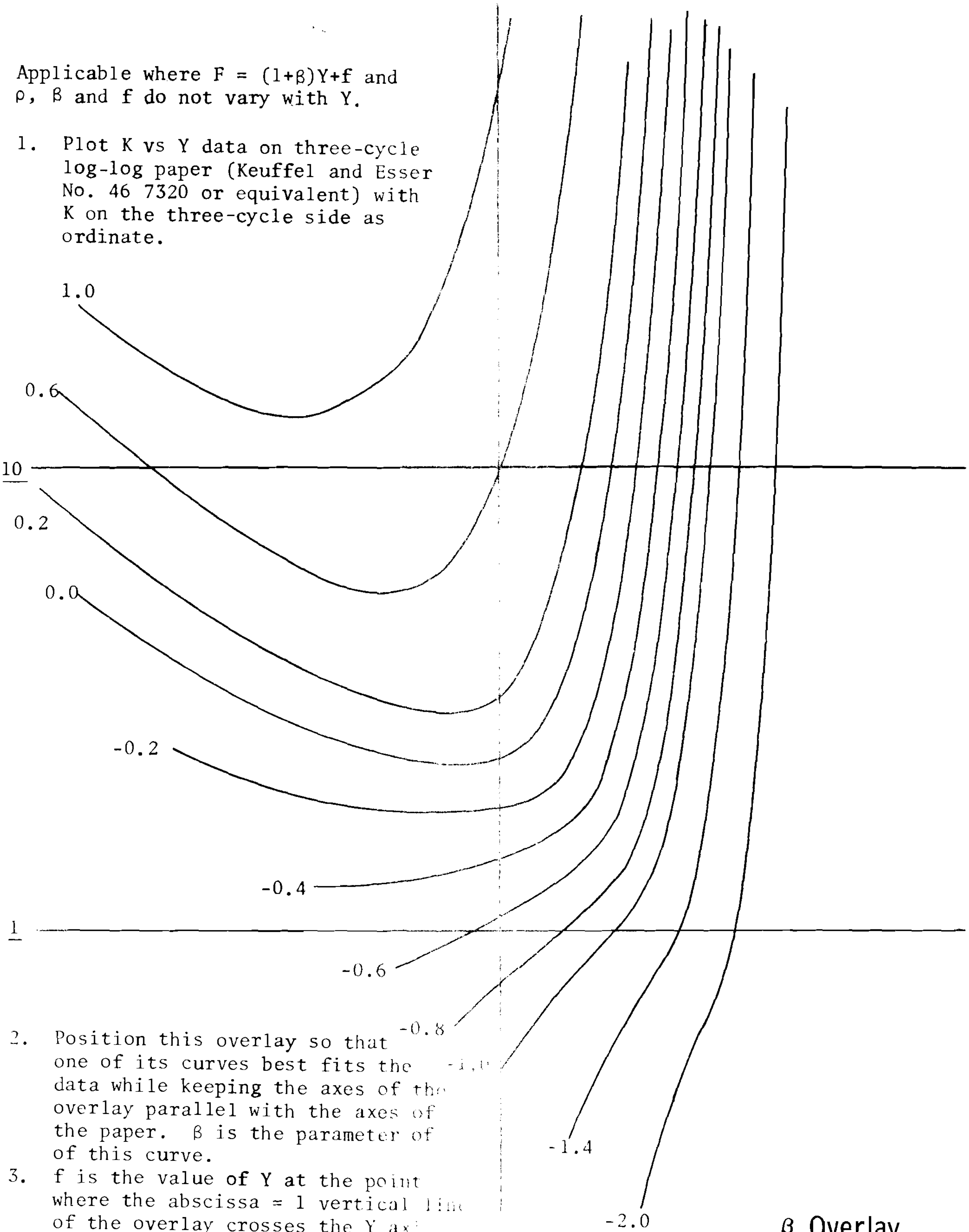
2. Position this overlay so that its curve best fits the data while keeping the axes of the overlay parallel with the axes of the paper.

3. F is the value of Y at the point where the abscissa = 1 line of the overlay crosses the Y axis.

4. F $\sqrt{\pi\rho}$ is the value of K where the ordinate = 1 line of the overlay crosses the K axis.

Applicable where $F = (1+\beta)Y+f$ and ρ, β and f do not vary with Y.

1. Plot K vs Y data on three-cycle log-log paper (Keuffel and Esser No. 46 7320 or equivalent) with K on the three-cycle side as ordinate.



2. Position this overlay so that one of its curves best fits the data while keeping the axes of the overlay parallel with the axes of the paper. β is the parameter of of this curve.
3. f is the value of Y at the point where the abscissa = 1 vertical line of the overlay crosses the Y axis.
4. $f\sqrt{\pi\rho}$ is the value of K where the ordinate = 1 horizontal line of the overlay crosses the K axis.

β Overlay